CSGB

CENTRE FOR **STOCHASTIC GEOMETRY**
AND ADVANCED **BIOIMAGING**

Annual Report 2016

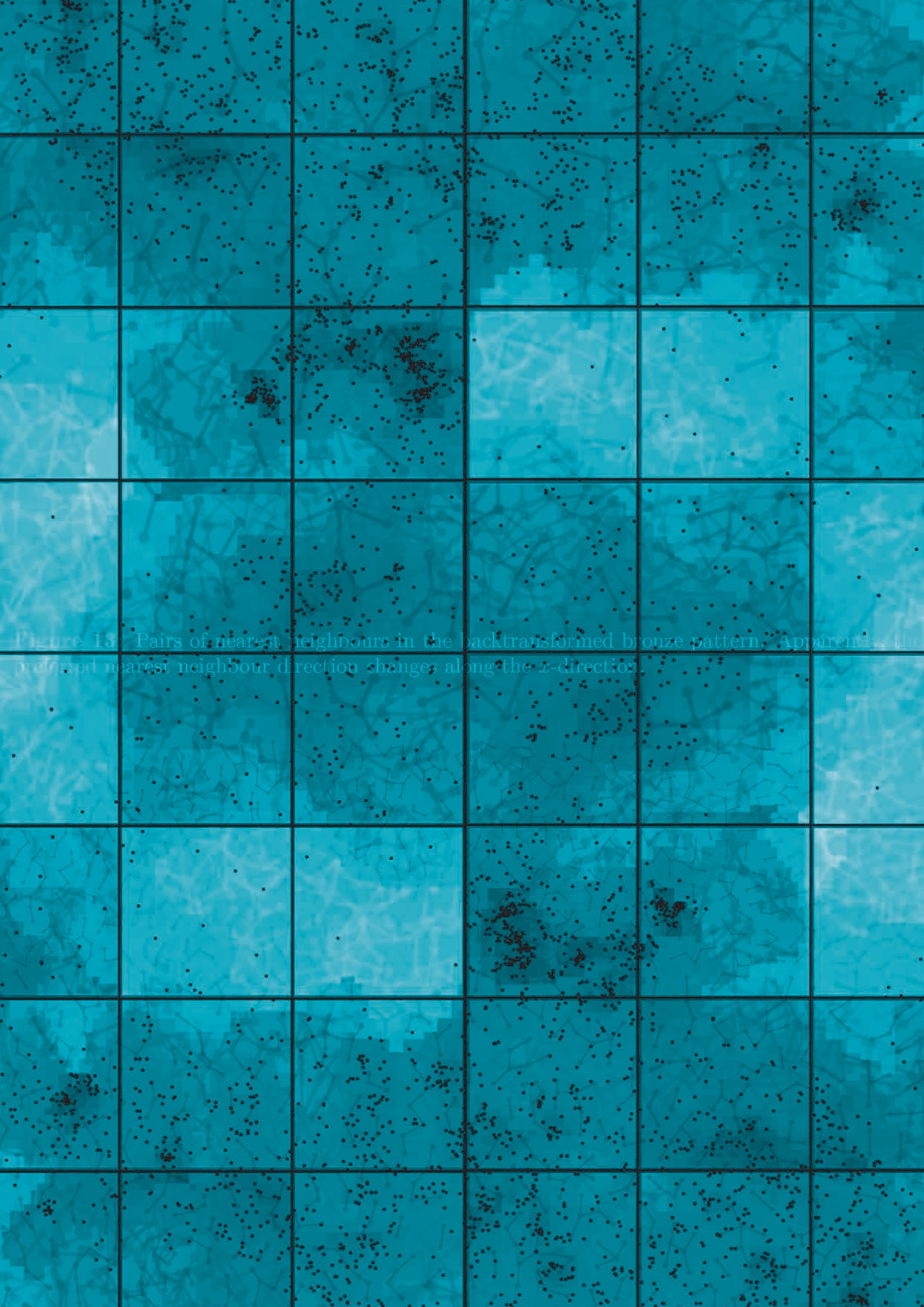

Figure 13: Pairs of nearest neighbours in the backtransformed bronze pattern. Apparently, the preferred nearest neighbour direction changes along the x-direction.

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Annual Report 2016, published April 2017

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INTRODUCTION

Centre for Stochastic Geometry and Advanced Bioimaging (CSGB) is a VKR Centre of Excellence, supported by the Villum Foundation. A main task is to advance the discipline of stochastic geometry, and thereby laying the mathematical foundations for novel methods of analyzing advanced bioimaging data. At the same time, we take up the concrete challenge of new types of bioimaging data.

CSGB has now been running for seven years. Since the beginning in April 2010, CSGB has functioned as an inter-institutional collaboration between the Universities of Aarhus, Aalborg and Copenhagen. Four research groups participate in CSGB: the stochastic geometry group (AU), the section for stereology and microscopy (AU), the spatial statistics group (AAU) and the image section (KU). In 2016, the stereology and microscopy group was restructured and is expected in 2017 to move to Aarhus University Hospital, as part of a new Core Centre for Molecular Morphology.

2016 has been a year of changes for CSGB. Four PhD students that started in the first funding period of CSGB (1 April 2010 – 31 March 2015) successfully defended their thesis. Furthermore, during 2016 and the beginning of this year, ten new PhD students started at CSGB (pp. 12-13). Three of the PhD students were recruited from abroad. During 2017, we expect to hire one or two more PhD students. These young researchers constitute the new generation of PhD students that are financed by the grant from the Villum Foundation in the second funding period of CSGB (1 April 2015 – 31 March 2020).

A highlight in 2016 was the finalizing of the volume, edited by Markus Kiderlen and myself, entitled *Tensor Valuations and their Applications in Stochastic Geometry and Imaging*, to be published April 2017 in the Springer Lecture Notes in Mathematics Series (p. 15). The volume contains 15 chapters, written by experts in their fields, and provides a comprehensive overview of the modern theory of tensor valuations. The volume includes new yet unpublished results that make this volume an up-to-date survey of valuation theory. Chapters on applications of tensor valuations deepen understanding and emphasize the usefulness of theoretical concepts.

Research-wise, 2016 was a year of many advances within the six work packages that form the research plan of the second funding period of CSGB. A detailed account for each work package is given in this annual report (pp. 20-31). An overview of the research results in 2016 (p. 18) and a description of collaborative projects (p. 19) are also provided. A highlight in 2016 was the publication in the top journal *Biometrika* of the theoretical paper on point processes with linear structures. The motivation for constructing these stochastic geometry models comes from a need of analyzing the minicolumn hypothesis in neuroscience.

During 2016, we arranged a number of international workshops and PhD courses. A highlight was here the *Workshop on Geometry and Stochastics of Nonlinear, Functional and Graph Data*, 15 – 19 August, Rønne, Denmark. This workshop was arranged jointly by the stochastic geometry group, AU, and the image section, KU. We managed to attract international top researchers within each of the fields nonlinear statistics, functional data analysis, and random topology and graphs (pp. 36-37). The venue was Green Solution House which is located in the middle of a large park in the outskirts of Rønne. The building and landscape present the latest developments in the field of sustainable architecture.

April 2017
Eva B. Vedel Jensen

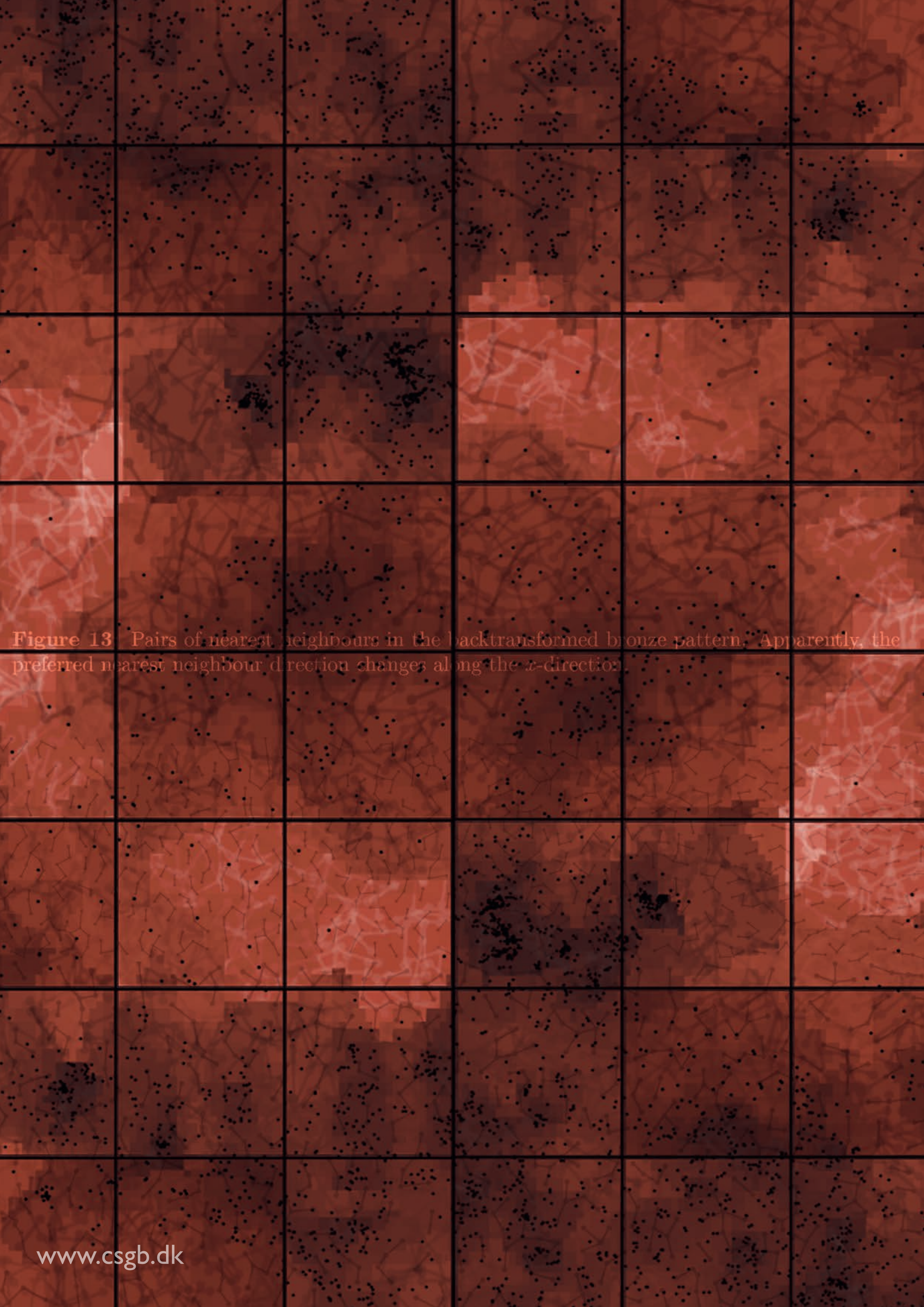


Figure 13 Pairs of nearest neighbours in the backtransformed bronze pattern. Apparently, the preferred nearest neighbour direction changes along the x-direction.

ORGANIZATION AND STAFF

ORGANIZATION AND STAFF

STAFF

Centre for Stochastic Geometry and Advanced Bioimaging - CSGB - is an interdisciplinary research collaboration between

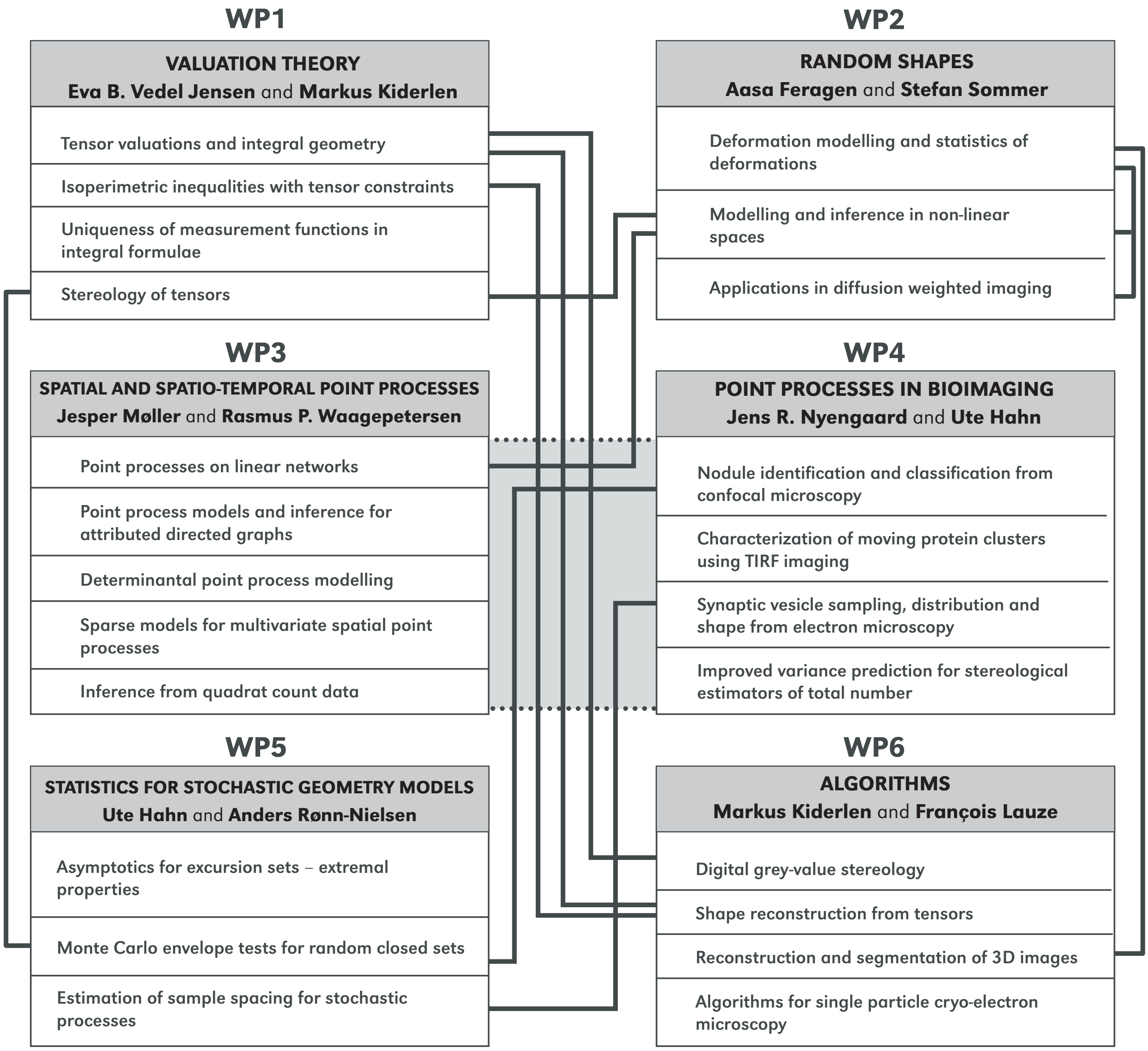
- Stochastic Geometry Group, Department of Mathematics, AU
- Section for Stereology and Microscopy, Department of Clinical Medicine, AU
- Spatial Statistics Group, Department of Mathematical Sciences, AAU
- Image Section, Department of Computer Science, KU

The four participarting research groups are in the following denoted by **AU-math**, **AU-bio**, **AAU** and **KU**, respectively.

The staff consisted in 2016 of 5 professors, 14 associate professors, 3 assistant professors, 4 postdocs and 12 PhD students, see page 44-45 for details.

RESEARCH PLAN

The research of CSGB is in the second funding period (1 April 2015 – 31 March 2020) organized in six work packages, see the research diagram below that also shows the many interconnections between the projects. The principal investigators of the work packages are also shown in the diagram, just below the title of the work packages.



NEWS ABOUT STAFF

Christophe Biscio (AAU)
During 2016, Christophe Biscio was appointed assistant professor at Department of Mathematical Sciences, Aalborg University. He has a PhD degree from Nantes in September 2015 and immediately after, he was hired as postdoc in Aalborg. Christophe is participating in the CSGB work package **Spatial and spatio-temporal point processes**. His focus is topological data analysis, with a view to applications to tree structures and spatial point processes.

Sune Darkner (KU)
In 2016, Sune Darkner was promoted from assistant to associate professor at Department of Computer Science, University of Copenhagen. He has been a member of the CSGB staff since the very beginning in 2010. In the second funding period of CSGB (2015-2020), he is participating in the **Random shapes** work package. Sune is part of the team developing shortest-path tractography algorithms in diffusion weighted imaging.

Aasa Feragen (KU)
Aasa Feragen has in 2016 obtained additional funding from the Villum Foundation and the Lundbeck Foundation. In the first case, the funding covers a two-year postdoc position. The project title is *Learning discriminative embeddings of big complex data*. In the second case, a three-year PhD study on *Image registration with topological differences through stratified diffeomorphisms* is funded. Aasa is one of the principal investigators of the **Random shapes** work package.

Ege Rubak (AAU)
Ege Rubak is on leave November 2016 – May 2017 from his associate professorship position in Aalborg. He works as guest researcher at Curtin University, Perth. Ege has a very fruitful collaboration with professor Adrian Baddeley, Curtin University. Together with Rolf Turner, they published in 2015 the monograph *Spatial Point Processes: Methodology and Applications with R*. Ege is participating in the CSGB work package **Spatial and spatio-temporal point processes**.

Stefan Sommer (KU)
In January 2016, Stefan Sommer joined the editorial board of the *Journal of Mathematical Imaging and Vision* (JMIV) as associate editor. Together with Aasa Feragen, Stefan is principal investigator of the **Random shapes** work package. Stefan and Aasa were both very active in the organization of the *Workshop on Geometry and Stochastics of Nonlinear, Functional and Graph Data*, 15 – 19 August 2016, Green Solution House, Rønne, see page 36-37.

ORGANIZATION AND STAFF

APPOINTMENTS 2016/2017

Abdel-Rahman Al-Absi (AU-bio)
Abdel-Rahman Al-Absi has a joint master degree in neuroscience from Bordeaux II University and Alexandria University. In March 2017, he started as PhD student at AU-bio. Part of the PhD study concerns 3D reconstruction of neuronal dendritic trees and spines. The analysis of these data, that can be represented as marked point processes, involves collaboration with the AAU team and fall under the work package **Point processes in bioimaging**.

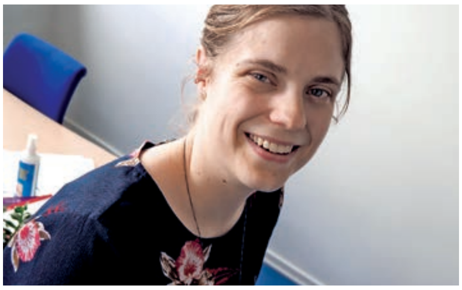
Ina Trolle Andersen (AU-math)
Ina Trolle Andersen obtained her PhD degree in September 2016 with a thesis on statistical inference for microscopy data. The thesis describes efficient sampling procedures and modelling by spatial point processes. Immediately after, she was hired at CSGB as postdoc at AU-math. Ina is participating in the work package **Point processes in bioimaging**. The focus is on modelling and statistical inference on PALM microscopy data.

Heidi Søgaard Christensen (AAU)
Heidi Søgaard Christensen graduated from Department of Mathematical Sciences, Aalborg University, in June 2016. Since September 2016, she has been a PhD student at AAU. She is part of the team working with **Spatial and spatio-temporal point processes**. One of the focus points of her PhD study is point processes on graphs. In 2015, she spent one semester at Department of Statistics, University of Auckland, New Zealand.

Andreas Dyreborg Christoffersen (AAU)
Andreas Dyreborg Christoffersen obtained his master degree from Department of Mathematical Sciences, Aalborg University, in June 2016. Andreas has been a PhD student at AAU since September 2016. His PhD project concerns **Point processes in bioimaging**, more specifically, marked point processes for 3D anisotropic point patterns and their applications to the minicolumn hypothesis in neuroscience. In 2015, he spent one semester at Department of Statistics, University of Auckland, New Zealand.

Rikke Krogh Eriksen (AU-math)
Rikke Krogh Eriksen started in August 2016 at AU-math immediately after finished bachelor studies in a special five year PhD stipend, co-financed by the Faculty of Science and Technology, AU. Rikke is participating in the work package **Valuation theory**. The focus is on Minkowski tensors and set-valued means in stochastic geometry and integral geometry. Her bachelor thesis concerned C*-algebras.

Line Kühnel (KU)
Line Kühnel has a master degree in mathematical statistics from Department of Mathematical Sciences, University of Copenhagen. She started her PhD study at the KU group in January 2016. Her PhD project is part of the **Random shapes** work package and concerns modelling and statistical inference for data on manifolds. One particular type of model class is regression models.



Nick Yin Larsen (AU-bio)
Nick Yin Larsen obtained in 2016 a master degree in neuroscience and neuroimaging from University of Chinese Academy of Science in collaboration with Aarhus University and Sino Danish Center. From March 2017, Nick has been a PhD student at AU-bio. His PhD project involves concrete studies of minicolumn structure and volume tensors of neurons in the human brain. The project is important for the work package entitled **Valuation theory** as well as the work package **Point processes in bioimaging**.

Anton Jussi Olavi Mallasto (KU)
Anton Jussi Olavi Mallasto graduated from Aalto University, Finland, with a thesis entitled *Lie Groups and Applications to Shape Analysis*. Since November 2016, he has been a PhD student at the KU group. Anton is participating in the **Random shapes** work package. He has a strong background in algebra and geometry. Anton spent an exchange semester in KAIST, South Korea, during fall 2015 as part of his master studies.

Francisco Andrés Cuevas Pacheco (AAU)
Francisco Andrés Cuevas Pacheco has a master degree from Federico Santa Maria Technical University in Chile. Since March 2016, he has worked as a PhD student at the AAU group. He is participating in the work package **Spatial and spatio-temporal point processes**. His special focus is on spatial point processes on the sphere and in time.

Mads Stehr (AU-math)
Mads Stehr has a bachelor degree in mathematical statistics from Department of Mathematical Sciences, University of Copenhagen. In 2016, he decided to move to Aarhus to start a PhD study at AU-math. Mads is enrolled in the so-called 4+4 PhD programme, with start February 2017. Passing a midterm exam, he will obtain his master degree. His PhD project lies mainly within the work package **Valuation theory**.

Helene Svane (AU-math)
Helene Svane obtained her master degree from Department of Mathematics, University of Aarhus, in June 2016. Her master thesis lies within topology. Helene started her PhD studies in August 2016 at AU-math. Her PhD project continues a PhD project from the first funding period of CSGB. The aim is to study to what extend topology and geometry can be reconstructed from 3D digital images. The project lies within the work package **Algorithms**.

ORGANIZATION AND STAFF

From the PhD defense of Ina Trolle Andersen.

PHD DEFENSES

The following PhD students at CSGB defended their thesis in 2016:

February 2016 | **Katrine Hommelhoff Jensen** (KU)
Statistical methods for single-particle electron microscopy

September 2016 | **Ina Trolle Andersen** (AU-math)
Statistical inference for microscopy data – efficient sampling and modelling by spatial point processes

September 2016 | **Astrid Kousholt** (AU-math)
Minkowski tensors: stereological estimation, reconstruction and stability results

November 2016 | **Sabrina Tang Christensen** (AU-math) | *Reconstruction of topology and geometry from digitisations*

COLLABORATION

The collaboration between Alexis Arnaudon, Darryl Holm and Stefan Sommer on stochastic deformation models was continued in 2016.

Some of the joint reconstruction and segmentation work were presented at a meeting in Lund on 12 December 2016. It was agreed to build up a semi-formal collaboration around tools for image processing for synchrotron tomography.

The image section is participating as partners in a project that aims at developing mathematical models and image analysis algorithms for realizing the full potentials of the newly established MAX IV synchrotron in Lund, Sweden. The project is funded by the Capital Region of Denmark Health Research Fund. The project partners will by the end of the project establish a knowledge center for clinical synchrotron image formation.

ESTABLISHMENT OF NEW NETWORK

Sune Darkner and Jon Sporring from the image section is participating in the establishment of the Danish BioImaging Network (www.danishbioimaging.dk). The Danish BioImaging (DBI) Network gathers the BioImaging community in Denmark with the aim to strengthen BioImaging as an essential tool in life science, by promoting and facilitating collaboration, knowledge exchange, best practice and bioimaging research infrastructures within the medical and natural sciences, nationally and internationally.

Sune Darkner, Mads Nielsen, Akshay Pai and Stefan Sommer have received a network grant to establish international collaboration focusing on investigation of Lewy Body and Alzheimers Dementia. A first visit to Cleveland Clinic, Ohio, has been made.

SPRINGER LECTURE NOTES IN MATHEMATICS

A volume in the Springer Lecture Notes in Mathematics Series

Markus Kiderlen and Eva B. Vedel Jensen (eds.)

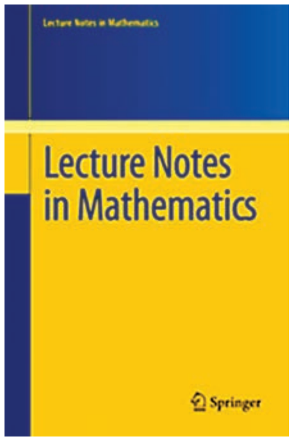
Tensor Valuations and their Applications in Stochastic Geometry and Imaging

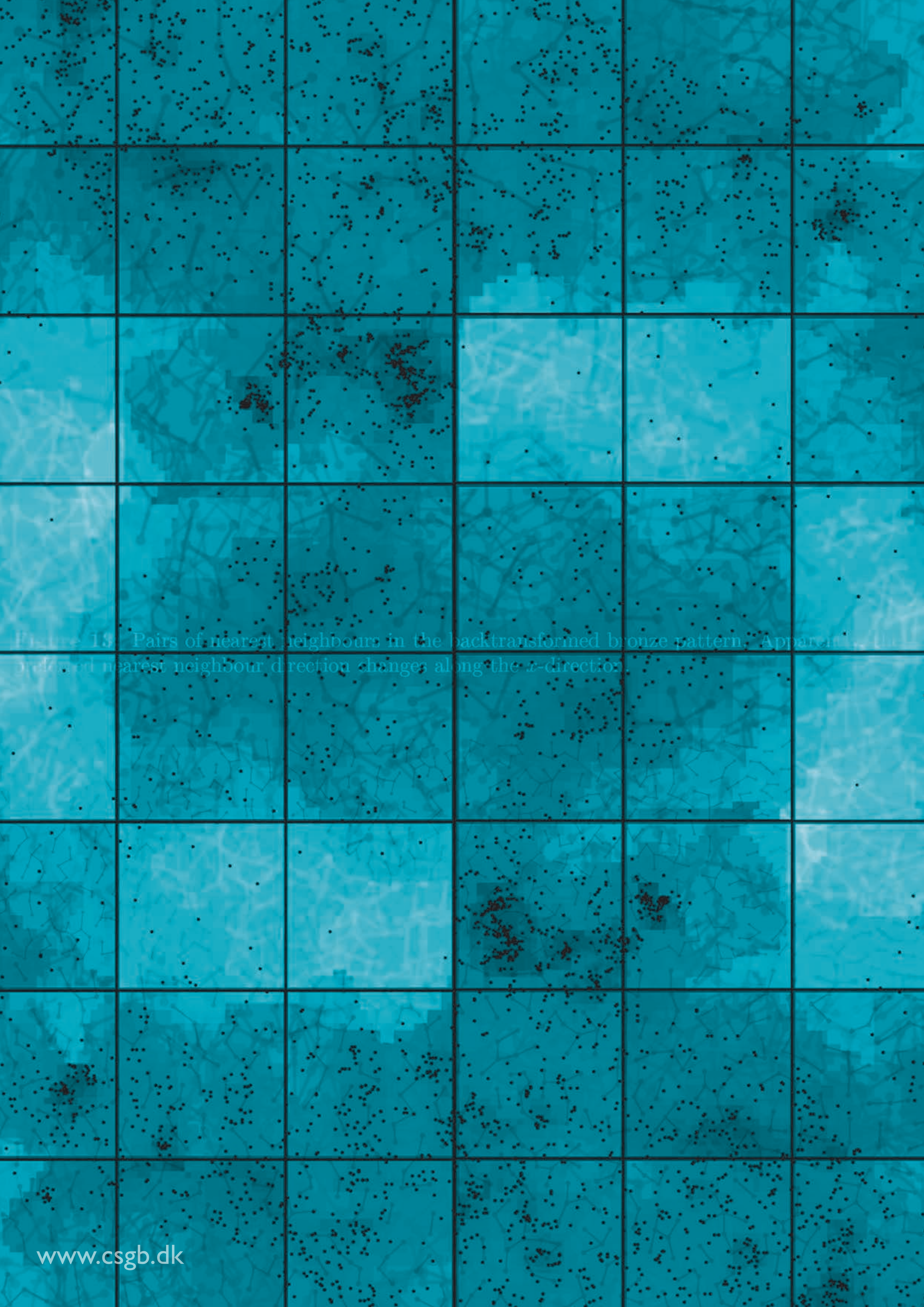
has been edited during 2016. In 15 chapters, written by experts in their fields, a comprehensive overview of the modern theory of tensor valuations is given. The volume includes new yet unpublished results that make this volume an up-to-date survey of valuation theory. Chapters on applications of tensor valuations deepen understanding and emphasize the usefulness of theoretical concepts.

Starting with classical results concerning scalar-valued valuations on the families of convex bodies and convex polytopes, the volume proceeds to the modern theory of tensor valuations. Product and Fourier-type transforms are introduced and various integral formulae are derived. New and well-known results are presented, together with generalizations in several directions, including extensions to the non-Euclidean setting and to non-convex sets. A variety of applications of tensor valuations to models in stochastic geometry, to local stereology and to imaging are also discussed.

The collaboration with the DFG funded research group *Geometry and Physics of Spatial Random Systems* has played a very important role for the concept and content of this book.

CONTENTS

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RESEARCH

RESEARCH

RESEARCH OVERVIEW 2016

In the second funding period of CSGB, the research is organized in six work packages

WP1: Valuation theory
WP2: Random shapes
WP3: Spatial and spatio-temporal point processes
WP4: Point processes in bioimaging
WP5: Statistics for stochastic geometry models
WP6: Algorithms

Below, we give an overview of the research taken place in 2016 within each of these work packages. On the following page, we describe four collaborative projects. The general aim of the collaborative projects is to develop new quantitative bioimaging analysis methods based on tools from stochastic geometry. A detailed description of the research results obtained in 2016 within each of the six work packages may be found on page 20-31.

In the work package **Valuation theory**, one of the main results obtained in 2016 concerns a new rotational Crofton formula for Minkowski tensors. The formula shows how rotational averages of intrinsically defined Minkowski tensors on sections passing through the origin are related to the geometry of the sectioned set. The formula is valid for sets of positive reach. A similar formula, derived in CSGB I for convex sets, now appears as a special case of the new formula. During 2016, a new stereological estimator of volume tensors from vertical sections has also been derived.

The aim of the work package **Random shapes** is to advance the statistical modelling of random shapes that reside in non-linear spaces such as manifolds and stratified spaces. During 2016, new models for stochastic variation of shape have been developed. One important approach has been to directly incorporate stochasticity into the shape model by using stochastic coadjoint motion in the Lie algebra of the diffeomorphism group, acting on the shape space. The problem of parameter estimation in these non-linear stochastic models has been addressed. Another important class of models, studied in 2016, is regression models for data on a manifold.

In the work package **Spatial and spatio-temporal point processes**, the research on point processes on graphs has been intensified during 2016. New covariance models depending only on geodesic distance have been developed in the case of tree graphs, and the models have been extended to any graph with Euclidean edges by replacing the geodesic distance with the resistance metric. A next step is to construct Gaussian and log Gaussian processes on graphs. In 2016, we have also succeeded in developing a new one-dimensional functional summary statistic that under mild conditions contains the full information of the persistence diagram.

The work package **Point processes in bioimaging** involves a number of collaborative projects some of which are described on the next page. During 2016, the project on protein clusters advanced by the development of a new point process model for PALM microscopy data. This type of data, consisting of point patterns of fluorescent protein molecules, are very challenging to analyze because some of the molecules are not observed and those observed may have multiple appearances with noisy positions. The point proces model takes these phenomena into account. Non-parametric inference as well as parametric inference, based on moments, and Bayesian inference are under development.

One of the main achievements in 2016 within the work package **Statistics for stochastic geometry models** was the establishment of new results for the extremal behaviour of non-Gaussian random fields, modelled as integrals with respect to a so-called Lévy basis. More specifically, we obtained results on the asymptotic size of excursion sets for such random fields. These results are given in terms of the asymptotic probability that the excursion set contains an object of a given size. The object may be a ball of a given radius or a line segment of a given length.

The work package **Algorithms** involves the development of computer-intensive algorithms, including a study of their mathematical and statistical properties. In 2016, a focus point has been shape reconstruction from tensors. A general algorithm for reconstruction of convex bodies in n-dimensional space from finitely many surface tensors has been developed. Furthermore, it has been shown that a convex body is determined up to translation by finitely many surface tensors if and only if the body is a polytope. Shape reconstruction from volume tensors has also been considered. During 2016, new efficient algorithms for reconstruction of particles, from noisy data obtained by cryo-electron microscopy, have been implemented.

COLLABORATIVE PROJECTS

The **AU-bio group** plays a key role in the collaborative projects, providing a forum for testing and validating methods developed in collaboration with the three other participating research groups. PhD students and postdocs as well as senior CSGB researchers participate in these projects.

Stereology of tensors by optical microscopy (AU-bio/AU-math)

The aim is to further develop and implement in optical microscopy new stereological methods of estimating Minkowski tensors. Stereology for tensor-valued functionals has a totally different character than stereology for scalar-valued functionals and require clever 3D sampling. Implementation in optical light microscopy has been developed for the cases where it is difficult to construct 3D voxel images in optical microscopy, using the optical rotator probe. During 2016, a new stereological estimator of mean particle volume tensors from vertical sections has been constructed. A next step is to implement the associated simpler vertical sampling design in optical microscopy. A particular challenge is to find sampling strategies for the estimation of volume tensors of neurons in the human brain.

The minicolumn hypothesis (AU-bio/AAU)

The minicolumn hypothesis in neuroscience claims that neurons and other brain cells have a columnar arrangement, perpendicular to the surface of the brain. A new tool, the cylindrical *K*-function, and a new model, the Possion line cluster point process, have been developed by CSGB researchers for investigating this hypothesis, considering 3D point patterns for the locations of (pyramidal and non-pyramidal) neurons. The challenge is now to extend our models and methods to more complicated data such as two-point pattern data sets of neurons which will provide a better basis for testing the minicolumn hypothesis. The more complicated data can be described by marked point processes.

Neuronal dendritic trees and spines (AU-bio/AAU)

During 2016, 3D methods of reconstructing dendritic trees have been developed. The reconstructions are obtained, using a *z*-stack of optical sections. In the reconstruction, the soma, the axon and the whole dentritic tree of pyramidal neurons are identified. The small membranous protrusions on the dendrites, the so-called spines, are also identified. The spines represent the post-synaptic part of the synapse and they receive input from the presynaptic part originating from other neurons. The next step is to develop spatial marked point process models for pyramidal neurons, describing the relationship between characteristics of the dendritic trees and the spines. One possibility is point processes on graphs.

Synaptic vesicles by FIB-SEM (AU-bio/AAU/KU)

A local FIB-SEM imaging technique is the foundation for further studies of synaptic vesicles at CSGB, based on spatial point process modelling. At AU-bio, we have in 2016 strived to improve the image quality and are now able to produce images of a sufficient quality to identify vesicles and other organelles. FIB-SEM offers a possibility for isotropic voxels and with a distance of down to 5nm between consecutive imaged surfaces, multiple sections through individual vesicles assist the distinction of vesicles in close spatial approximation. A next step is now to make 3D reconstructions of positions of vesicles and other organelles and use these as input in spatial point process models.

Other collaborations

Within CSGB, there is also direct collaboration between senior researchers. Examples concern the AU-math/KU collaborative project on estimation of **sample spacing** in stochastic processes and the AU-math/AU-bio collaborative project on algorithms for single particle **cryo-electron microscopy**. Yet another example concerns collaboration between the AU-math and KU-groups, concerning **envelope testing** of shortest-path tractography in diffusion weighted imaging.

WP1

Researchers
Karl-Anton Dorph-Petersen
Rikke Krogh Eriksen
Eva B. Vedel Jensen
Markus Kiderlen
Astrid Kousholt
Jens R. Nyengaard
Andrew du Plessis
Ali Hoseinpoor Rafati
Anne Marie Svane
Johanna F. Ziegel

Valuation theory

Tensor valuations are tensor-valued additive functionals on the family of convex or more general sets. Special cases are the classical intrinsic volumes (volume, surface area, length, Euler number) which are Minkowski tensors of rank zero. In stochastic geometry, the focus has in recent years turned to Minkowski tensors of rank one or higher which provide information about position, orientation and shape of spatial structures.

In 2016, **integral geometry of tensor valuations** has been developed further by CSGB researchers. Motivated by applications in local stereology, a new rotational Crofton formula has been derived for Minkowski tensors (Svane & Jensen, 2016). The formula, valid for **sets of positive reach**, concerns the integral mean of intrinsically defined Minkowski tensors on *j*-dimensional **linear subspaces**, see Figure 1. In the special case of rank 0 tensors, the formula reduces to the main result in Jensen & Rataj (2008). Results for Minkowski tensors of rank (*r*, *s*) with *s*=0, obtained in Auneau-Cognacq *et al.* (2013) for convex sets, are also special cases of the main result in Svane & Jensen (2016).

The formula derived in Svane & Jensen (2016) shows how rotational averages of intrinsically defined Minkowski tensors on sections passing through the origin are related to the geometry of the sectioned set. In particular, for Minkowski tensors of order $j-1$ on *j*-dimensional linear subspaces, an explicit formula for the rotational average involving hypergeometric functions is derived. Sectioning with lines and hyperplanes through the origin is considered in detail.

The case where the sections are not restricted to pass through the origin is also studied in Svane & Jensen (2016). Here, Crofton formulae for the integral mean of intrinsically defined Minkowski tensors on *j*-dimensional **affine subspaces** are obtained for sets of positive reach. Such formulae were first derived in Hug *et al.* (2008, Theorem 2.5 and 2.6) in the case of convex sets. The integrals can be expressed as linear combinations of Minkowski tensors. However, the coeeficients in these linear combinations were represented as iterated sums and thus very complicated to evaluate. Recently, in Hug & Weis (2016), the results have been generalized to tensor-valued curvature measures and the constants have been simplified. Our main contribution, relating to affine subspaces, is the generalization of the formulae to sets of positive reach.

During 2016, a new **stereological estimator of mean particle volume tensors** from vertical sections has been developed in Kousholt *et al.* (2016). The new method is useful in cases where it is difficult to construct 3D voxel images of the particles. The estimator uses instead observations in planar sections through a sample of the particles. The new estimator is constructed for a particle process which can be represented as a stationary marked point process $\{[x_i; K_i]\}$. Here, x_i is a reference point of the *i*th particle $x_i + K_i$, and K_i is a compact set containing the origin *o*. Under the assumption that the particle mark distribution is invariant under rotations around a fixed axis, called the **vertical axis**, it is possible to construct a ratio-unbiased (and consistent in a probabilistic sense) estimator of the mean particle volume tensors, based on observations on vertical sections through a sample of particles. Basically, the estimator involves **area tensors**, determined in the vertical sections. In a simulation study with Lévy particles, see Kousholt *et al.* (2016), the new estimator has a superior behavior compared to an earlier estimator based on observations in several optical planes (Rafati *et al.*, 2016). The new estimator has great potential use in optical microscopy.

In 2016, the editing of a volume in the Springer Series, *Lecture Notes in Mathematics*, has been finalized, see Kiderlen & Jensen (2017). This volume gives an up-to-date introduction to tensor valuations and their applications. Starting with classical results concerning scalar-valued valuations on the families of convex bodies and convex polytopes, it proceeds to the modern theory of tensor valuations. Product and Fourier-type transforms are introduced and various integral formulae are derived. New and well-known results are presented, together with generalizations in several directions, including extensions to the non-Euclidean setting and to non-convex sets. A variety of applications of tensor valuations to models in stochastic geometry, to local stereology and to imaging are also discussed. Jensen & Kiderlen (2016), Kousholt *et al.* (2016) and Svane (2016) appear as chapters in this volume.

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Svane, A.M. & Jensen, E.B.V. (2016): Rotational Crofton formulae for Minkowski tensors and some affine counterparts. *CSGB Research Report* **2016-13**. Submitted.

$$
\begin{aligned}
&\int_{\mathcal{L}_j^d} \Phi_{k,L}^{r,s}(X \cap L)\,\mathrm{d}L \\
&= \frac{1}{r!s!\sigma_{j-k+s}} \int_{\operatorname{nor} X} \frac{x^r}{|x|^{d-j}} \sum_{|I|=j-1-k} \frac{\prod_{i\in I} \kappa_i(x,n)}{\prod_{i=1}^{d-1} \sqrt{1+\kappa_i(x,n)^2}} \\
&\quad \times \int_{\mathcal{L}_{j-1}^{x\perp}} (\pi(n|L^x))^s \frac{\mathcal{G}(L^x, A_I(x,n))^2}{|p(n|L^x)|^{j-k}}\,\mathrm{d}L\,\mathcal{H}^{d-1}(\mathrm{d}(x,n))
\end{aligned}
$$

Figure 1:
Rotational integral formula, showing how rotational averages of intrinsically defined Minkowski tensors on sections passing through the origin are related to the geometry of the sectioned set. For details, see Svane & Jensen (2016).

WP2

Researchers
Sune Darkner
Aasa Feragen
Ute Hahn
Line Kühnel
François Lauze
Matthew Liptrot
Anton Mallasto
Mads Nielsen
Jens R. Nyengaard
Andrew du Plessis
Stefan Sommer
Anne Marie Svane

Random shapes

In this work package we study random shapes that reside in non-linear spaces. An important example is the **tree-spaces**, arising naturally in modelling of anatomical networks. The field of non-linear statistics has close connections to functional data analysis, and random topology and graphs.

In 2016, results of the random shapes work package included new models for **stochastic variation of shapes**, and new results on the relationship between curvature, data variation and statistical inference. The focus has been on exploring the relation between geometry and statistics, and the results on stochastics in non-linear spaces point to a range of future theoretical research directions and practical applications of the theory on empirical data.

In Arnaudon *et al.* (2016a, b), a method is introduced for introducing stochasticity in models of data that are affected by **deformations of an underlying domain**. Such data appear in the field of computational anatomy. A typical example concerns human organ shapes under the influence of disease processes where analysis of longitudinal shape changes are important for the study of e.g. Alzheimer's disease. Shapes and shape changes are inherently non-linear, and statistical analysis of shape variation must take this non-linearity into account. In classical shape models, shapes are considered deterministic objects, and shape changes are modelled deterministically. However, random variation occurs in observed shapes, e.g. in bioimaging.

The developed model allows stochasticity to be directly incorporated in the shape model by using **stochastic coadjoint motion** in the Lie algebra of the diffeomorphism group. The model preserves many properties of the rich geometric structure present in the deterministic setting: Dynamics appear as optimal solutions for a stochastically pertubed variational problem, and the deterministic Euler-Poincaré equation has a stochastic counterpart.
The stochastic diffeomorphisms generate corresponding stochastic motion for a range of different shape data types through the action of the diffeomorphism group on the shape spaces.

An important problem is **parameter estimation** in these non-linear stochastic models that couple to the underlying geometry. We have developed estimation methods by approximating the Fokker-Planck equations of the transition density of the stochastic process. Maximum likelihood estimation is obtained using Monte-Carlo sampling of bridge processes. This is used in Arnaudon *et al.* (2016a) to estimate the noise in shapes of human corpora callosa.

In more general non-linear spaces, non-isotropic random variation couples with the curvature of the space resulting in nonholonomic constraints. This can be modelled, using the frame bundle of the manifold. In Sommer & Svane (2016), the **frame bundle geometry** is described, short-time asymptotics for the transition density is derived and sample estimators for mean and covariance are suggested. The paper Sommer (2016) further explored the geometry behind most probable stochastic paths for a driving semi-martingale that through stochastic development is mapped to a manifold.

In Kühnel & Sommer (2016), a regression model is derived, based on this framework that extends Euclidean linear regression. Non-linear statistical models have been used (Kühnel *et al.*, 2016) to separate warp and intensity effects in images. For an illustration, see Figure 1.

In Feragen & Fuster (2016), classical and recently proposed Riemannian metrics and interpolation schemes on the space of **symmetric positive definite** (SPD) matrices are reviewed. Several metrics are considered, including the Frobenius, Wasserstein, Fisher Rao and log Euclidean metrics. This review contains important results, valid for analysis of tensorial data in general. With the paper, a software package with MATLAB scripts is released.

Four papers, relating to **diffusion weighted imaging** (DWI), have been published in 2016, see Kasenburg *et al.* (2016a-d). The majority of the research, on which these papers are based, was already described in *CSGB Annual Report* **2015**.

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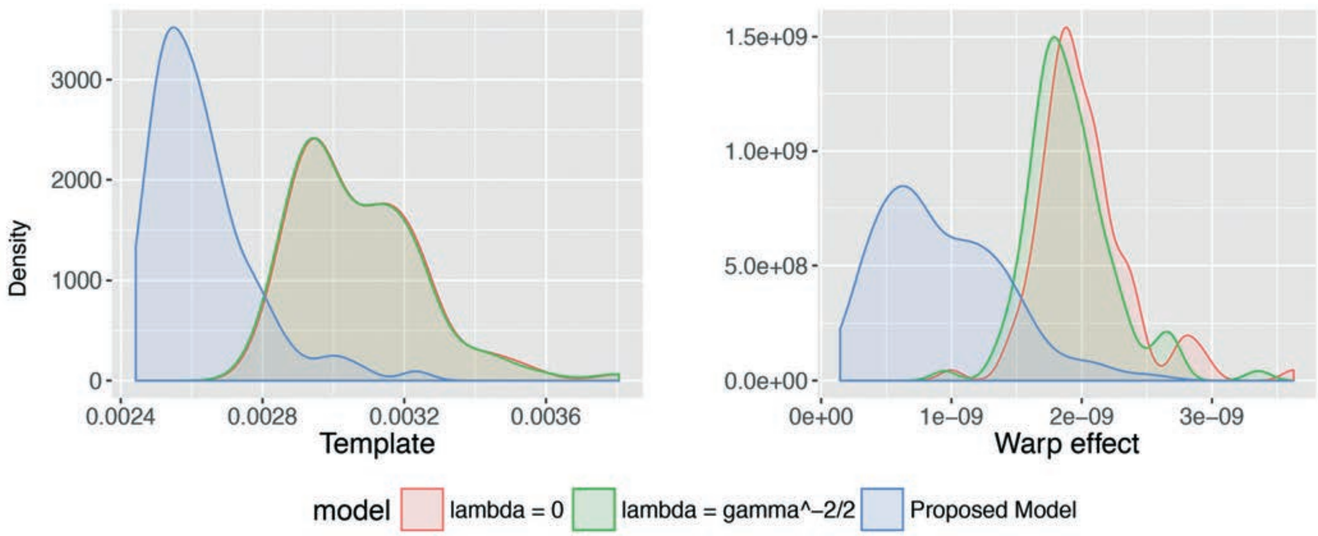

Figure 1:
Density plots for the mean squared differences of template and warp estimates for the three models. The plot to the left shows the density for the mean squared difference for the template effect and the plot to the right shows the mean squared difference for the warp effect. For details, see Kühnel *et al.* (2016).

WP3

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Spatial and spatio-temporal point processes

Below, we focus on some of the new developments taken place in 2016. The CSGB researchers associated with this work package have also during 2016 contributed decisively to the work package WP4 on point processes in bioimaging, see page 26-27.

Point processes are the fundamental building blocks of stochastic geometry models. Modelling and statistical analysis of point processes in Euclidean spaces is very well developed. However, there is need for transferring this methodology to **non-Euclidean** spaces such as **linear networks** and **directed graphs**.

During 2016, Jakob G. Rasmussen and Jesper Møller's collaboration with Ethan Anderes, University of California at Davis, on **random fields and point processes on graphs** with Euclidean edges has been intensified. Particularly, new covariance models depending only on geodesic distance have been developed in the case of tree graphs, and the models have been extended to *any graph with Euclidean edges* by replacing the geodesic distance with the resistance metric. This will be of importance when constructing Gaussian processes and log Gaussian Cox processes on graphs with Euclidean edges. This work is expected to result in a long paper for a top journal such as Annals of Statistics, and it will be continued by including the newly started PhD student Heidi Søgaard Christensen.

An analogue of the **likelihood ratio test** for spatial Gibbs point process models fitted by maximum pseudolikelihood or maximum composite likelihood is investigated in Baddeley *et al.* (2016). Adjustments developed for composite likelihoods of finite systems of random variables are adapted to the point process setting in order to obtain an asymptotic chi-square distribution under the null hypothesis. Recent results in point process theory are used to estimate the composite information and sensitivity from data.

A **persistent diagram** is a multiset of points in the plane describing the persistence of topological features of a compact set when a scale parameter varies. Since statistical methods are difficult to apply directly on persistence diagrams, various alternative **functional summary statistics** have been suggested, but either they do not contain the full information of the persistence diagram or they are two-dimensional functions. Biscio & Møller (2016) suggest a new functional summary statistic that is one-dimensional and hence easier to handle, and which under mild conditions contains the full information of the persistence diagram. Its usefulness is illustrated in various statistical settings concerned with **point clouds** and **brain artery trees**, see the example in Figure 1.

Jacobsen *et al.* (2016) investigate how the choice of spatial point process for **generating random sampling patterns** affects the numerical stability of non-uniform generalized sampling between Fourier bases and Daubechies scaling functions. They consider binomial, Poisson and determinantal point processes and demonstrate that the more regular point patterns from the determinantal point process are superior. See also Jacobsen & Møller (2016).

Møller & Waagepetersen (2017) review developments in statistics for spatial point processes obtained within roughly the last decade. These developments include new classes of spatial point process models, parametric inference based on various types of estimating equations, maximum likelihood inference and Bayesian inference as well as non-parametric inference and new approaches to simulation based inference.

During 2016, the research on Takacs-Fiksel estimation (Coeurjolly *et al.*, 2016), Palm distributions (Coeurjolly *et al.*, 2017a, b), **determinantal point processes** (Biscio & Lavancier, 2017; Møller & Rubak, 2016) and **multivariate point processes** (Waagepetersen *et al.*, 2016) has been published or accepted for publication. See also Lavancier & Møller (2016) and Møller *et al.* (2016). The majority of this research has been described in *CSGB Annual Report* **2015**.

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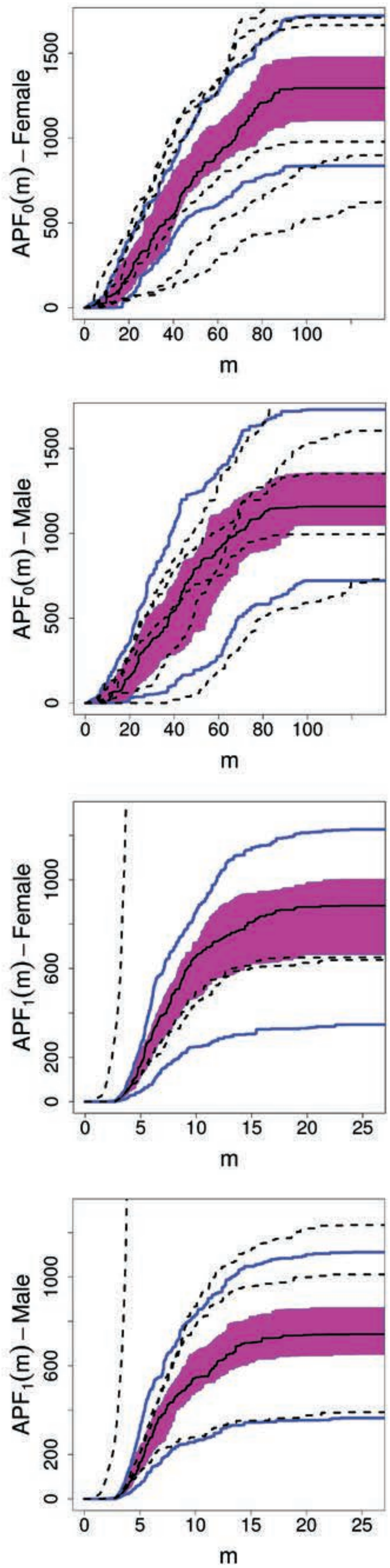

Figure 1:
Functional boxplots of accumulated persistence functions (APFs) for females and males obtained from the brain artery trees dataset (upper-lower): APF_0^F, APF_0^M, APF_1^F, APF_1^M. The dashed lines show the outliers detected by the 1.5 criterion. For more details, see Biscio & Møller (2016).

WP4

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Point processes in bioimaging

In 2016, one of the absolute highlights was the publication in *Biometrika* of the theoretical paper Møller *et al.* (2016) on point processes with linear structures. To detect anisotropy in point patterns with linear structures, in particular in the case of a columnar structure, this paper introduces a functional summary statistic, the **cylindrical *K*-function**, which is a directional *K*-function whose structuring element is a cylinder. Furthermore, a class of anisotropic Cox point processes, called Poisson line cluster point processes, is introduced. The points of such a process are random displacements of Poisson point processes defined on the lines of a Poisson line process. Parameter estimation for this model based on moment methods or Bayesian inference is developed in the case where the underlying Poisson line process is latent.

The motivation for constructing these models comes from bioimaging. The models can thus be used for analyzing the **minicolumn hypothesis** in neuroscience, which claims that pyramidal and other brain cells have a columnar arrangement, perpendicular to the surface of the brain (Rafati *et al.*, 2016). The next step is to use marked point processes in relation to the minicolumn hypothesis in neuroscience. Andreas Dyreborg Christoffersen started in September 2016 as a PhD student working on this research topic.

In relation to the project on **protein clusters**, we have during 2016 developed a point process model for PALM (Photo Activated Localization Microscopy) data. This type of data, consisting of point patterns of fluorescent protein molecules, are very challenging to analyze because some of the molecules are not observed and those observed may have multiple appearances with noisy positions. The point proces model takes these phenomena into account. Non-parametric inference as well as parametric inference, based on moments, and Bayesian inference are under development (Andersen *et al.*, 2017).

In Khanmohammadi *et al.* (2015), a novel three-dimensional marked point process model has been proposed by CSGB researchers for studying spatial interactions between **synaptic vesicles**. 3D representions of synapses obtained from a set of publicly available FIB-SEM images were used in the analysis. FIB-SEM (Focused Ion Beam Scanning Electron Microscopy) facilitates automated acquisition of large, complete image series for three-dimensional reconstructions. A prerequisite for reliable reconstruction of synapses is a clear visualization of individual synaptic vesicles with outer diameters generally in the range of 35-45 nm (Qu *et al.*, 2009).

The FIB-SEM imaging technique will provide the foundation for further studies based on spatial point process modelling, involving CSGB researchers from the image section, KU, the stereology and microscopy section, AU, and the spatial statistics group, AAU. This requires that we are able locally to produce high-resolution FIB-SEM images of sufficient quality for the 3D reconstructions. In 2016, we have strived to improve the image quality and are now able to produce images of an acceptable quality at 65.000 x magnification, which aids the identification of vesicles and other organelles considerably, see Figure 1. Our current efforts are directed towards increasing the contrast of the images - especially between the vesicle membranes and the surrounding cytoplasm – while maintaining the grey tones, in an attempt to move towards automatic annotation of vesicles. Furthermore, there is a great need for increased knowledge about the resolution of images in the *z*-axis. FIB-SEM offers a possibility for isotropic voxels and with a distance of down to 5 nm between consecutive imaged surfaces, multiple sections through individual vesicles assist the distinction of vesicles in close spatial approximation.

In 2016, we have initiated a collaborative project between CSGB researchers from the stereology and microscopy section, AU, and the spatial statistics group, AAU. The aim is to develop spatial marked point process models for pyramidal neurons, describing the relationship between characteristics of the dendritic tree and the spines which are small membranous protrusions on the dendrites. During 2016, methods of reconstructing in 3D the dendritic tree have been developed, see Figure 2.

During 2016, the research on **Matérn thinned Cox processes** has been published in Andersen & Hahn (2016). This research was motivated by the need for developing analysis tools for patterns of megakaryocytes in bone marrow.

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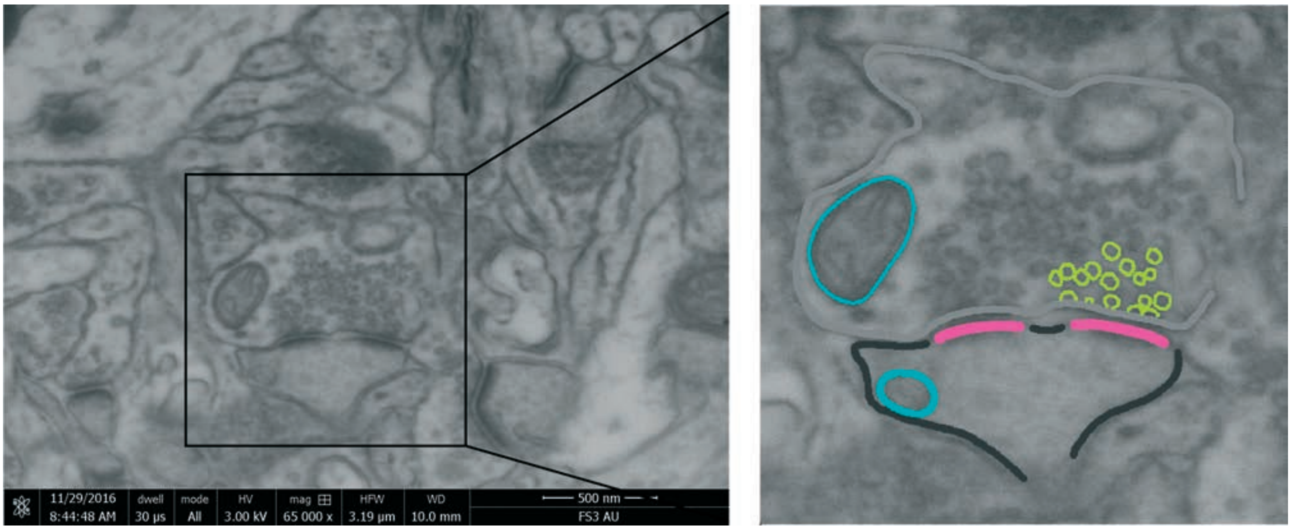
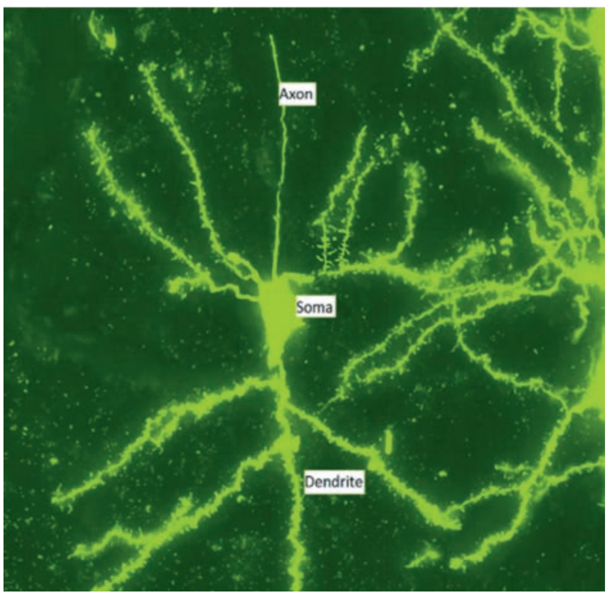


Figure 1
Left: Ultrastructure in the brain region medial prefrontal cortex of a mouse. The image was acquired using Focused Ion Beam Scanning Electron Microscopy. Magnification 65.000 x, resolution 1536 x 1024.
Right: Annotation of a synapse from the image to the left. The presynaptic axon (light grey), vesicles (green), a mitochondrion (blue), the post synaptic density (pink), and the postsynaptic dendrite (dark grey) are shown.

Figure 2
A 3D reconstruction of the soma, axon and whole dendritic tree of a pyramidal neuron from mice brain. The spines are small membranous protrusions on the dendrites that receive input from synapses from other neurons. The reconstruction has been obtained by Imaris software using a *z*-stack of optical sections, collected by newCAST software from Visiopharm, through the whole dendritic tree.



WP5

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Statistics for stochastic geometry models

The general aim of this work package is to develop new statistical inference procedures for stochastic geometry models. Typically, it is not possible to use classical statistical methods, based on e.g. likelihood functions, for the analysis of stochastic geometry models. The focus is on inference for random fields and spatial point processes, including analysis of functional data derived for these types of model classes. Inference for spatial point processes is also an important topic in work packages 3 and 4.

During 2016, we have continued studying the **extremal behaviour of random fields**, modelled as integrals with respect to a so-called Lévy basis. In Rønn-Nielsen & Jensen (2016a), it was shown that the supremum of a convolution equivalent random field has the same asymptotic tail distribution as the Lévy measure of the underlying Lévy basis. This result can be used in the analysis of brain image data, where it is of interest to detect the sites where a given field obtains values that are significantly large. However, it is needed to derive further results on the extremal behaviour of the field relating, for instance, to a collection of observations in a given neighbourhood of sites.

For this, it is relevant to consider the asymptotic behaviour of excursion sets (an excursion set for a random field is the set of sites, where the corresponding value of the field exceeds a certain level). In Rønn-Nielsen & Jensen (2016b), we have obtained results for the **asymptotic size of the excursion sets** by applying the techniques invented in Rønn-Nielsen & Jensen (2016a) for the asymptotic supremum results; now combined with equally involved geometrical considerations. We have computed the asymptotic probability that the excursion set contains a rotation of an object with a fixed radius. Examples of this object are a ball of a given radius and a line segment of a given length.

Another problem we have studied in 2016 is motivated by an application in electron microscopy, where ultra-thin microscopy sections are analyzed. For an illustration, see Figure 1. Here it is of interest to use information obtained from the observed values within sections to **estimate the distance** between two sections. In Rønn-Nielsen *et al.* (2016), we have proposed an estimator for this distance, for the case where the underlying random field is stationary and isotropic. Furthermore, we have derived an approximation to the variance of the estimator under the tractable, yet flexible, class of Lévy–based random field models, given by a kernel smoothing of a Lévy basis. Finally, we have examined the utility of the estimator and the approximate variance in simulation studies, see Figure 2, where it could be concluded that both perform very well. A future project will be to show consistency and asymptotic normality of the estimator under an expanding window regime.

During 2016, the research described in *CSGB Annual Report* **2015** on global envelope tests for spatial point processes (Myllymäki *et al.*, 2016), multiple Monte Carlo testing (Mrkvička *et al.*, 2016a) and Monte Carlo testing applied to spatial residuals (Mrkvička *et al.*, 2016b) has been published.

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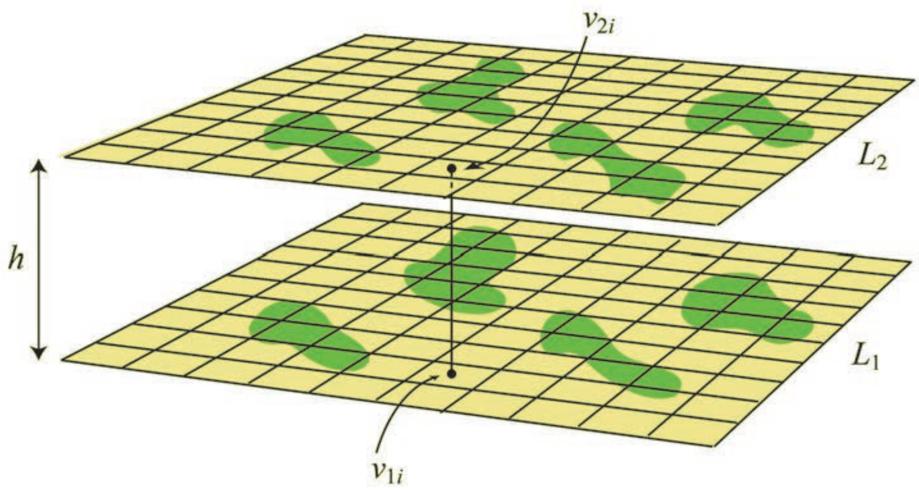
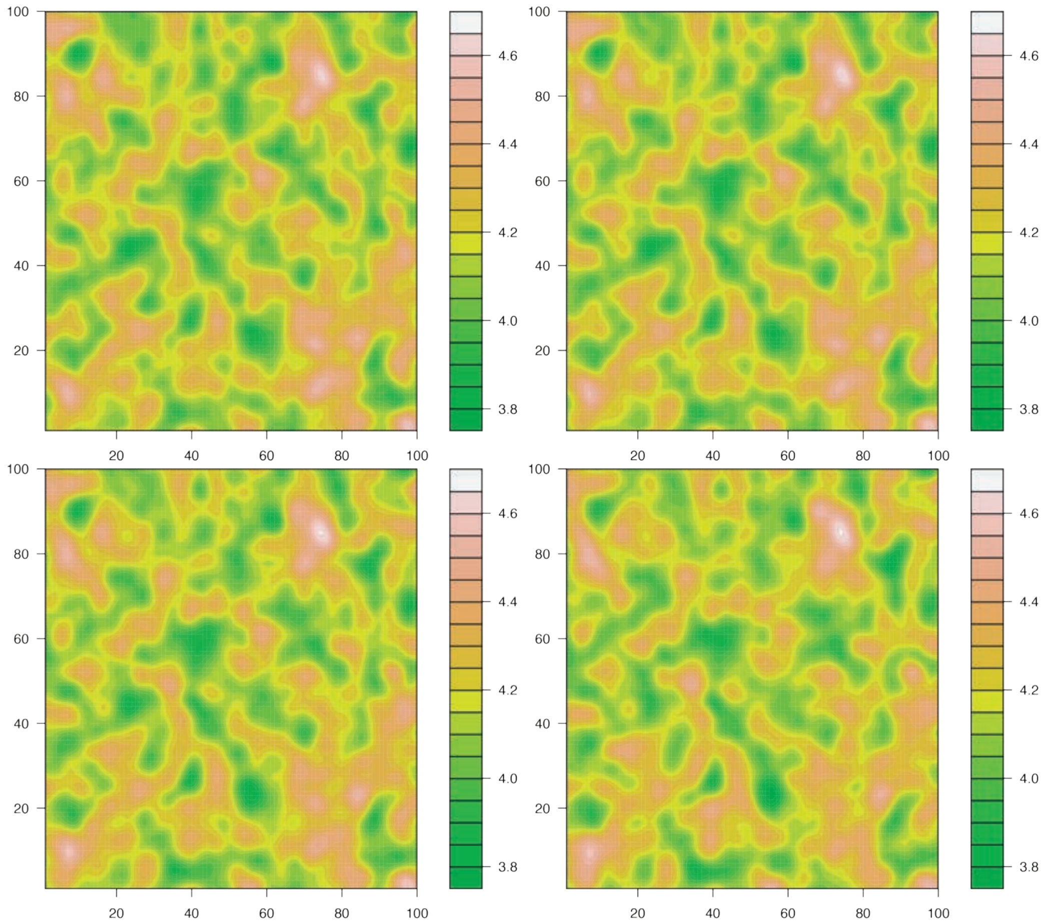


Figure 1: Illustration of the sampling of the random field on two parallel planes a distance *h* appart. For details, see Rønn-Nielsen *et al.* (2016).

Figure 2: Four jointly simulated parallel fields with NIG Lévy basis and a Gaussian kernel function. The three last fields are at distance 0.4, 1 and 2 from the first. For details, see Rønn-Nielsen *et al.* (2016).



WP6

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Algorithms

One of the modern tools for shape description are the **Minkowski tensors**, which also play a prominent role in work package WP1. In a statistical context, Minkowski tensors can, for instance, be used to diagnose anisotropy of a biological or physical structure. However, the particular detectable kind of anisotropy depends on the ranks of the tensors used for diagnosis. A ball and a regular cube with appropriate side-lengths have the same surface tensor of rank two due to their many symmetries, and can only be distinguished with the help of higher rank tensors. In the project on **shape reconstruction from tensors** we therefore continued to work on the 2D case (Kousholt & Kiderlen, 2016) to deepen the understanding of the information on the shape of an object when only finitely many Minkowski tensors are known.

Strikingly, **volume tensors and surface tensors** behave very differently. In Kousholt (2017), translation invariant surface tensors are considered, and it is shown that a convex body K is determined up to translation by finitely many of these tensors if and only if K is a polytope. The corresponding statement for volume tensors is treated in Hörrmann & Kousholt (2016). A weaker result is shown here: in the plane, convex sublevel sets of polynomials are uniquely determined by finitely many volume tensors, but it is not known if this condition is also necessary. In both cases (for volume tensors only in 2D) we derived a stability result: The Hausdorff metric of two (appropriately translated) convex bodies with the same tensors up to rank r decreases as a negative power of r. This was used to derive consistency results for newly developed reconstruction algorithms.

The algorithm for **reconstruction from surface tensors** is illustrated in Figure 1 for a three-dimensional example: the underlying set is the pyramid (Figure 1, upper left) and reconstruction from surface tensors up to rank 2, 3, and 4 is illustrated in Figure 1, upper right, lower left and right, respectively.

After we could show in the first funding period of CSGB (Svane, 2014) that local digital algorithms for extracting tensors from binary digital images are typically biased, we developed a **global digital algorithm** in Hug *et al.* (2016) for black/white images. It is based on a **Voronoi decomposition** of the pixel/voxel midpoints of the digitization and the inversion of a local Steiner formula. We showed that this algorithm yields asymptotically the correct tensors of a set of positive reach when the resolution of the digitization tends to infinity. That this algorithm is also valuable in the practical setting of final resolution was illustrated in Christensen & Kiderlen (2016), where it was implemented and applied to a realization of a Boolean model, see Figure 2, and other test sets. This paper also gave recommendations for the choice of the radii that are needed for the inversion of the Steiner formula.

Algorithms for **single particle cryo-electron microscopy** are very time- and resource-intensive due to the large amount of data that has to be processed simultaneously. Already at a two-dimensional level, classification is required to identify images that would be isometric copies of one another if they were noise-free. In the recent years, several expectation-maximization techniques have been proposed that aim at finding an approximate maximum likelihood estimate of the structures underlying a data set. We are currently implementing a reconstruction algorithm using principal component analysis for dimension reduction allowing to solve realistic 2D problems in a couple of hours on the GPU of an ordinary laptop.

The research on optimal algorithms for estimating specific intrinsic volumes from 3D Boolean models has been accepted for publication (Hörrmann & Svane, 2016). Two papers on **reconstruction and segmentation of 3D images** have been submitted during 2016, see Hansen & Lauze (2016) and Lauze *et al.* (2016).

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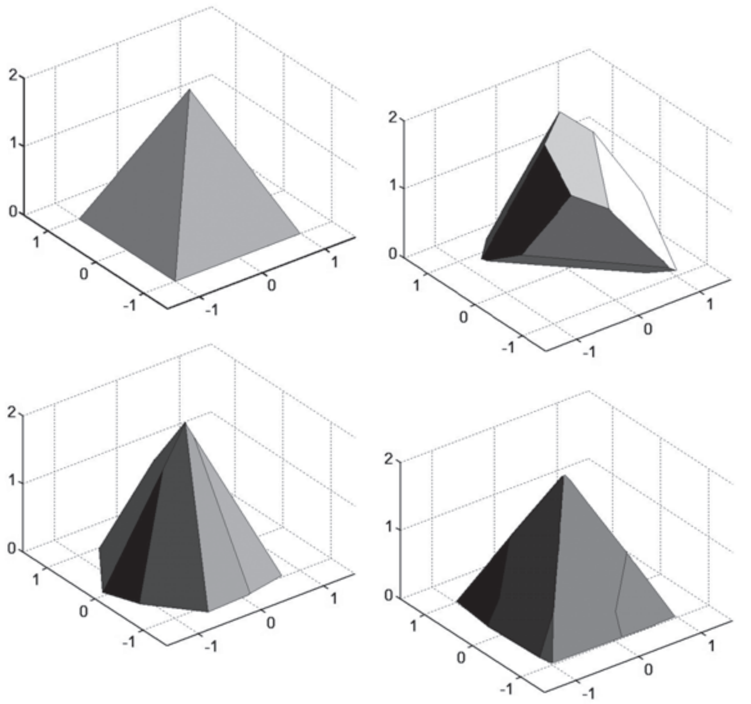


Figure 1:
Illustration of the developed algorithm for reconstruction from surface tensors. The underlying set is a pyramid (upper left) while reconstruction from surface tensors up to rank 2, 3 and 4 is shown in the upper right, lower left and lower right part of the figure, respectively. For details, see Kousholt (2017).

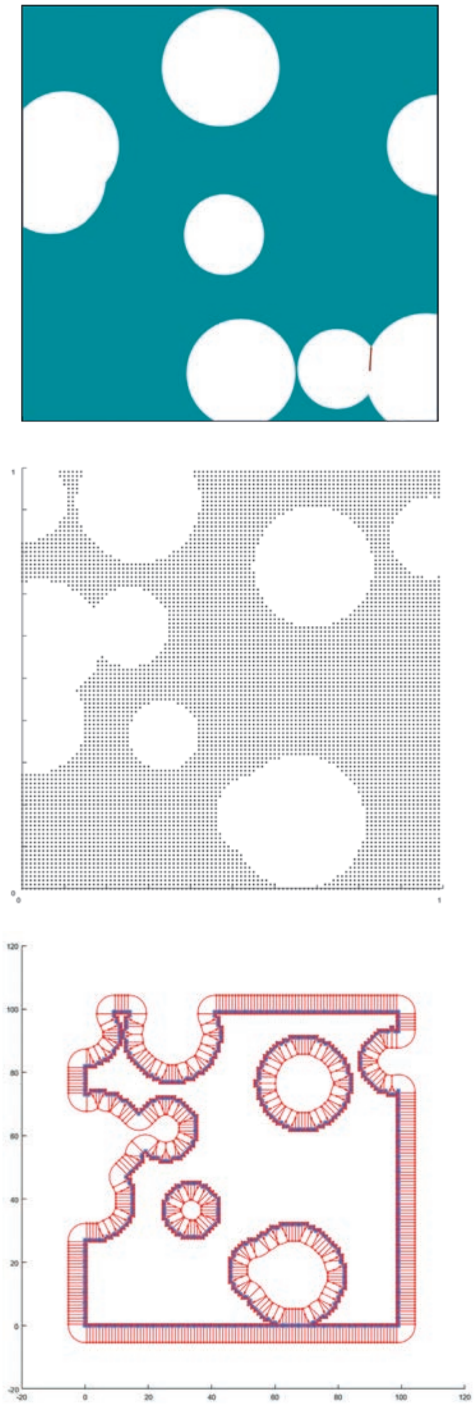


Figure 2:
The realization of the Boolean model (upper) that was used to test the Voronoi-based reconstruction of Minkowski tensors from digitizations of the background (middle). The *R*-bounded Voronoi cells (lower) required for the determination of generalized parallel volumes. For details, see Christensen & Kiderlen (2016).

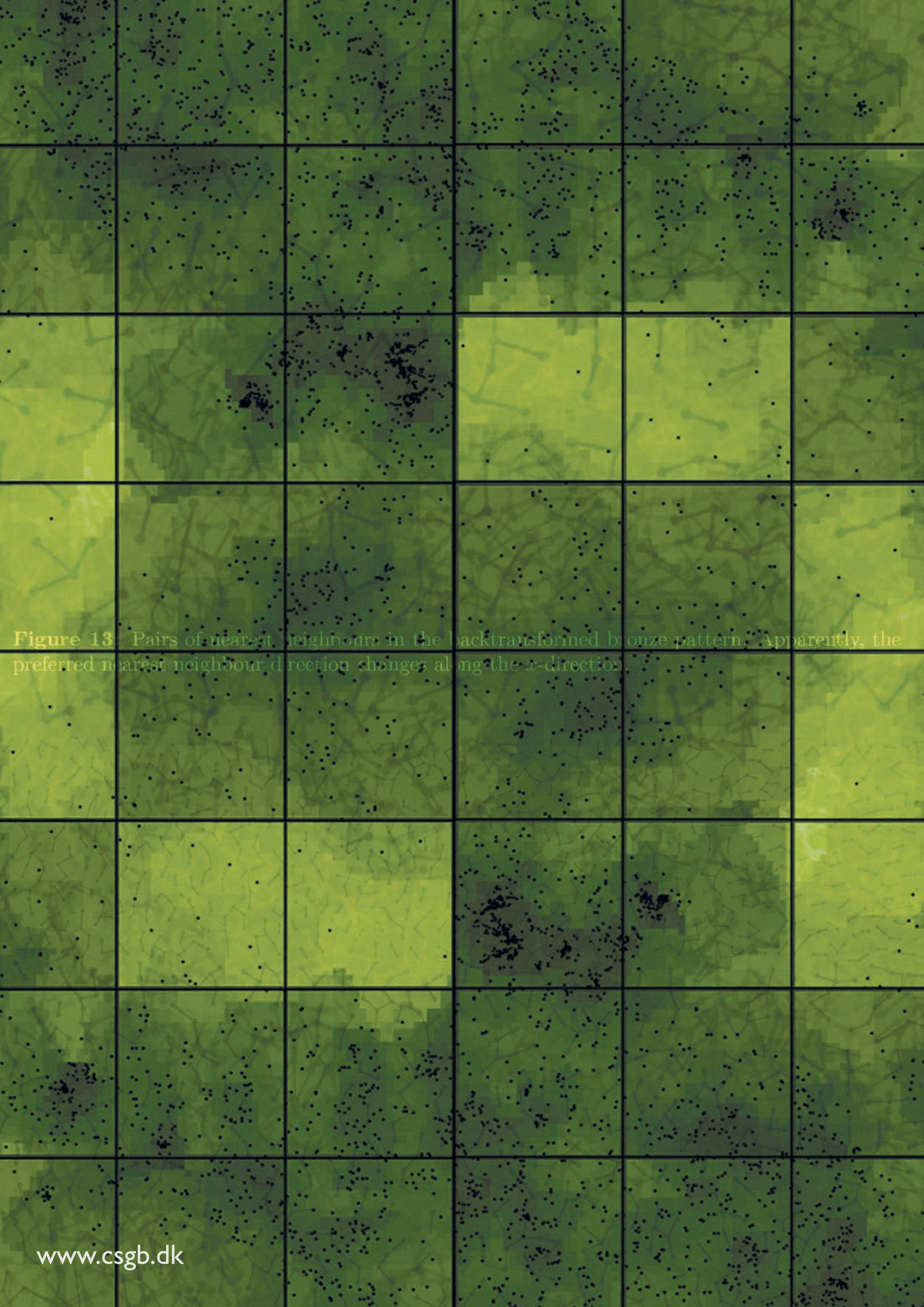


Figure 13 Pairs of nearest neighbours in the backtransformed bronze pattern. Apparently, the preferred nearest neighbour direction changes along the x-direction.

CENTRE ACTIVITIES

OVERVIEW – PAST AND PLANNED INTERNATIONAL ACTIVITIES

INTERNATIONAL CONFERENCES, SYMPOSIA AND WORKSHOPS

- Minisymposium on Inverse Problems for 3D Imaging
  3 May 2016, Copenhagen
- AU Workshop on Stochastic Geometry, Stereology and their Applications
  5 – 10 June 2016, Sandbjerg
- Stereology Workshop
  1 - 2 August 2016, Zaria
- Workshop on Geometry and Stochastics of Nonlinear, Functional and Graph Data
  15 – 19 August 2016, Rønne
- 6th International Conference on Scale Space and Variational Methods in Computer Vision
  4 – 7 June 2017, Kolding

INTERNATIONAL PHD COURSES

- Quantitative Medical Graphics
  6 and 13 April 2016, Aarhus
- Advanced Quantitative Neuro-Microscopical Course
  13-15 May 2016, Tehran
- Statistical Models and Methods for Spatial Point Processes
  15 and 22 June 2016, Peking
- Summer School on Semi-Supervised Learning
  8 - 12 August 2016, Slagelse
- Course on Neuroscience and Advanced Molecular Neurobiology
  18 - 26 September 2016, Beijing
- Stereology Course
  11 - 13 October 2016, Sandbjerg
- Statistical Models and Methods for Spatial Point Processes
  13 October 2016, Heidelberg/ 5 – 8 February 2017, Les Diablerets
- International Summer School on Graph Models
  14 – 18 August 2017, Tjarö



The excursion at the *Workshop on Stochastic Geometry, Stereology and their Applications* went to the northern part of Als in Denmark. The participants enjoyed the walk in the beautiful landscape and the coffee at the newly renovated Dyvig Badehotel.

AU WORKSHOP ON STOCHASTIC GEOMETRY, STEREOLOGY AND THEIR APPLICATIONS

5 – 10 JUNE 2016, SANDBJERG ESTATE, SØNDERBORG, DENMARK

Scope of the workshop
This interdisciplinary workshop brought together scientists from integral geometry, geometric probability, geometric measure theory, stochastic geometry, stereology, applied probability, spatial statistics and bioimaging. The scope of the workshop was to discuss recent theoretical advances in stochastic geometry and their potential for applications.
The workshop celebrated the scientific achievements of Eva B. Vedel Jensen.

Structure of the workshop
The scientific programme contained longer talks by keynote speakers. The workshop also had shorter invited and contributed talks as well as a poster session. The poster session was enhanced by short poster presentations in the plenum.

Organizers and financial support
The workshop was organized by Ute Hahn, Markus Kiderlen and Oddbjørg Wethelund. Financial support was obtained from the Aarhus University Research Foundation.

KEYNOTE SPEAKER (left to right)

Adrian Baddeley, Curtin University, Perth

Aasa Feragen, University of Copenhagen

Richard Gardner, Western Washington University

Wilfrid S. Kendall, University of Warwick

Jesper Møller, Aalborg University

Jens R. Nyengaard, Aarhus University

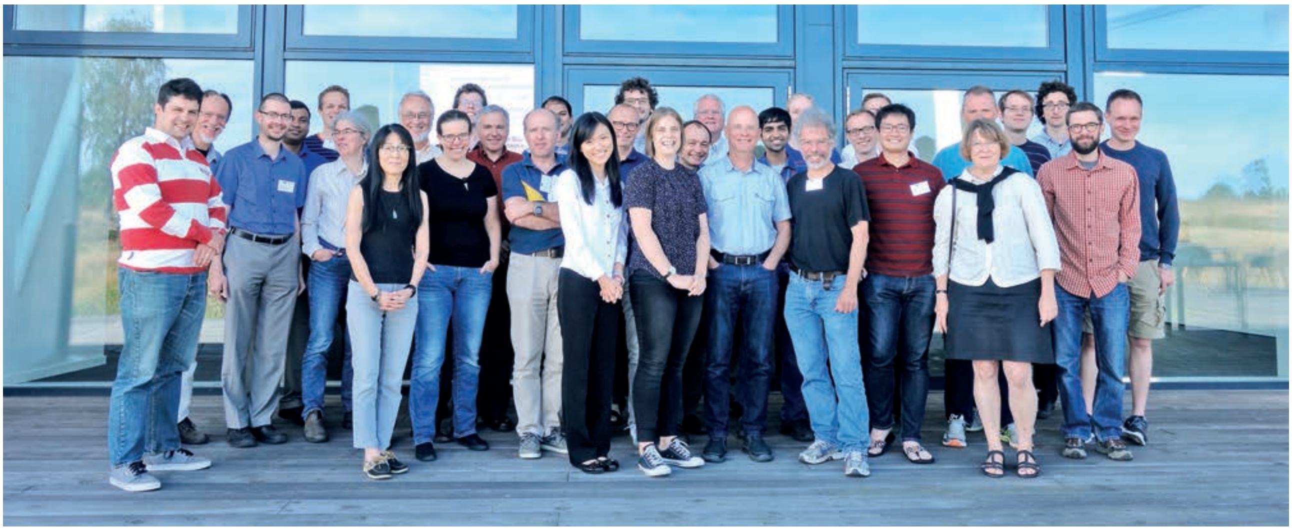
Michaela Prokešová, Charles University, Praha

Marie-Colette van Lieshout, Centre for Mathematics and Computer Science, Amsterdam

Wolfgang Weil, Karlsruhe Institute of Technology

WORKSHOP ON GEOMETRY AND STOCHASTICS OF NONLINEAR, FUNCTIONAL AND GRAPH DATA

15 – 19 AUGUST 2016, GREEN SOLUTION HOUSE, RØNNE, DENMARK



Scope of the workshop

The field of **nonlinear statistics** (NS) seeks to answer the fundamental questions that arise when defining new statistical models and tools in nonlinear spaces. The field spans from theoretical statistics and geometry to the development of data analysis tools that are directly applicable to actual data. A typical application area is bioimaging.

NS is a rapidly evolving research field. While there is a deep understanding of the mathematical and computational aspects of many data types living in nonlinear spaces, a detailed understanding of random variation in these spaces and how to handle randomness statistically is largely missing.

With this workshop, we reached out to key researchers in two closely related fields: **functional data analysis** (FDA), and **random topology and graphs** (RTG). The recent movement in FDA towards increasingly complex data structures has paved the way for a deeper statistical understanding of practical and commonly used methods in NS. The study of statistical properties of random graphs is closely related to NS through concepts like manifold learning, dimensionality reduction and topological data analysis. Complex topological properties of data living in nonlinear spaces, such as persistent homology, are studied in random topology.

The two fields FDA and RTG share problems and methodology, unresolved questions, and technical solutions with NS, and it is our belief, backed by world-leading experts in all three fields, that extended collaboration will lead to new results and development beyond the current state-of-the-art in these areas.

The venue

Green Solution House is located in Rønne on the island of Bornholm, in the middle of a large park, close to the beach. The design of the building is based on several parameters to show a holistic approach to sustainability. The building and landscape present the latest developments in the field of sustainable architecture.

The materials and products for Green Solution House meet or exceed high standards, prioritizing certifications and environmental labels, recyclability, social responsibily, use of resources, safety of compounds and energy in production, to name a few.

Organizers

The members of the scientific program committee were Aasa Feragen, Eva B. Vedel Jensen, Lars Lau Raket and Stefan Sommer. Oddbjørg Wethelund and Anya Kjærsig were in charge of the local organization.

INVITED SPEAKERS

Benjamin Eltzner
Institute for Mathematical Stochastics, Göttingen

Tom Fletcher
University of Utah

Thomas Hotz
TU Ilmenau

Stephan Huckemann
Institute for Mathematical Stochastics, Göttingen

Steve Marron
University of North Carolina at Chapel Hill

Hans-Georg Müller
University of California, Davis

Xavier Pennec
INRIA Sophia Antipolis

Mathew Penrose
University of Bath

Stephen M. Pizer
University of North Carolina at Chapel Hill

Anastasios Sidiropoulos
Ohio State University

Anuj Srivastava
Florida State University

Jonathan Taylor
Stanford University

Christoph Thäle
Ruhr Universität Bochum

Jane Ling Wang
University of California, Davis

Yu (Ryan) Yue
The City University of New York

Joseph Yukich
Lehigh University

INTERNAL CSGB WORKSHOPS

TWELFTH INTERNAL CSGB WORKSHOP
Sauntehus Slotshotel, Hornbæk, 19 - 20 May 2016

TALKS

- Abdel-Rahman Al-Absi, AU-bio: *3D reconstruction of neuronal dendritic trees and spines*
- Aasa Feragen/Sune Darkner, KU: *Assessing significance of estimated white-matter tracts: open problems*
- Ute Hahn, AU-math: *Point processes in ultra microscopy*
- Stine Hasselholt, AU-bio: *Practical application of FIB-SEM for brain imaging*
- Line Kühnel, KU: *Regression models for data on manifolds*
- Jesper Møller, AAU: *Palm distributions and functional summary statistics for point processes on the sphere*
- Andrew du Plessis, AU-math: *Some mathematics behind reconstruction*
- Björn Sander, AU-bio: *Class averaging in cryo-EM*
- Stefan Sommer, KU: *Stochastic evolution equations in geometric mechanics*
- Rasmus Waagepetersen, AAU: *Palm distributions for log Gaussian Cox processes*

THIRTEENTH INTERNAL CSGB WORKSHOP
HUSET, Middelfart, 3 - 4 November 2016

TALKS

- Ina Trolle Andersen, AU-math: *Spatial point process models for PALM microscopy*
- Christophe Biscio, AAU: *The accumulated persistence function, a new useful functional summary statistic for topological data analysis, with a view to brain artery trees and spatial point process applications*
- Aasa Feragen, KU: *Metrics on covariance matrices - are they useful and should we try to generalize them to Gaussian processes?*
- Christian Hauskov Iversen, AU-bio: *AutoRSA*
- Eva B. Vedel Jensen, AU-math: *Estimation of sample spacing*
- Line Kühnel, KU: *Regression models for data on manifolds*
- Jens R. Nyengaard, AU-bio: *Sampling and analysis of nerve fibres from corneal confocal microscopy*
- Ege Rubak, AAU: *Spatial point processes on the sphere*
- Björn Sander, AU-bio: *Classification of single-particle images by expectation maximization*



From the *Twelfth Internal CSGB Workshop*, 19 – 20 May 2016, Sauntehus Slothotel, Hornbæk.

CSGB RESEARCH REPORTS 2016

1. Biscio, C.A.N. & Møller, J. (2016): The accumulated persistence function, a new useful functional summary statistic for topological data analysis, with a view to brain artery trees and spatial point process applications. *CSGB Research Report* **2016-15**. Submitted.

2. Christensen, S.T. & Kiderlen, M. (2016): Comparison of two global digital algorithms for Minkowski tensor estimation. *CSGB Research Report* **2016-10**.

3. Feragen, A. & Fuster, A. (2016): Geometries and interpolations for symmetric positive matrices. *CSGB Research Report* **2016-14**. Submitted.

4. Jensen, E.B.V. & Kiderlen, M. (2016): Rotation invariant valutions. *CSGB Research Report* **2016-03**. To appear in *Lecture Notes in Mathematics* **2177**. Springer.

5. Hug, D., Kiderlen, M. & Svane, A.M. (2016): Voronoi-based estimation of Minkowski tensors from finite samples. *CSGB Research Report* **2016-06**. To appear in *Discrete Comput. Geom.*

6. Hörrmann, J. & Kousholt, A. (2016): Reconstruction of convex bodies from moments. *CSGB Research Report* **2016-07**. Submitted.

7. Hörrmann, J. & Svane, A.M. (2016): Local digital algorithms applied to Boolean models. *CSGB Research Report* **2016-05**. *Scand. J. Stat.*, in press. DOI 10.1111/sjos.12257.

8. Kousholt, A. (2016): Reconstruction of *n*-dimensional convex bodies from surface tensors. *CSGB Research Report* **2016-08**. Has appeared as *Adv. Appl. Math.* (2017) **83**, 115-144.

9. Kousholt, A., Ziegel, J.F., Kiderlen, M. & Jensen, E.B.V. (2016): Stereological estimation of mean particle volume tensors in $\mathbb{R}^3$ from vertical sections. *CSGB Research Report* **2016-09**. To appear in *Lecture Notes in Mathematics* **2177**. Springer.

10. Kühnel, L., Sommer, S., Pai, A. & Raket, L.L. (2016): Most likely separation of intensity and warping effects in image registration. *CSGB Research Report* **2016-16**. Submitted.

11. Møller, J. & Rubak, E. (2016): Determinantal point processes and functional summary statistics on the sphere. *CSGB Research Report* **2016-02**. Submitted.

12. Rønn-Nielsen, A. & Jensen, E.B.V. (2016): Excursion sets of infinitely divisible random fields with convolution equivalent Lévy measure. *CSGB Research Report* **2016-11**. Submitted.

13. Rønn-Nielsen, A., Sporring, J. & Jensen, E.B.V. (2016): Estimation of sample spacing in stochastic processes. *CSGB Research Report* **2016-17**. Submitted.

14. Sommer, S. (2016): Anisotropically weighted and nonholonomically constrained evolutions on manifolds. *CSGB Research Report* **2016-12**. Has appeared as *Entropy* (2016) 18, 425-445.

15. Sommer, S. & Svane, A.M. (2016): Modelling anisotropic covariance using stochastic development and sub-Riemannian frame bundle geometry. *CSGB Research Report* **2016-01**. To appear in *Journal of Geometric Mechanics*.

16. Svane, A.M. (2016): Valuations in image analysis. *CSGB Research Report* **2016-04**. To appear in *Lecture Notes in Mathematics* **2177**. Springer.

17. Svane, A.M. & Jensen, E.B.V. (2016): Rotational Crofton formulae for Minkowski tensors and some affine counterparts. *CSGB Research Report* **2016-13**. Submitted.

CSGB JOURNAL AND PROCEEDINGS PUBLICATIONS, BOOK CHAPTERS

Andersen, I.T. & Hahn, U. (2016): Matérn thinned Cox processes. *Spat. Stat.* **15**, 1-21.

Ardalan, M., Wegener, G., Polsinelli, B., Madsen, T.M. & Nyengaard, J.R. (2016): Neurovascular plasticity of the hippocampus one week after a single dose of ketamine in genetic rat model of depression. *Hippocampus* **26**, 1414-1423.

Baddeley, A., Turner, R. & Rubak, E. (2016): Adjusted composite likelihood ratio test for spatial Gibbs point processes. *J. Stat. Comp. Sim.* **86**, 922-941.

Boggild, S., Molgaard, S., Glerup, S. & Nyengaard, J.R. (2016): Spatiotemporal patterns of sortilin and SorCS2 localization during organ development. *BMC Cell Biology* **17**, 1-18.

Coeurjolly, J.-F., Guan, Y., Khanmohammadi, M. & Waagepetersen, R.P. (2016): Towards optimal Takacs-Fiksel estimation. *Spat. Stat.* **18**, 396-411.

Chen, F., du Jardin, K.G., Waller, J.A., Sanchez, C., Nyengaard, J.R. & Wegener, G. (2016): Vortioxetine promotes early changes in dendritic morphology compared to fluoxetine in rat hippocampus. *Eur. Neuropsychopharmacol.* **26**, 234-245.

Feragen, A. & Hauberg, S. (2016): Open problem: kernel methods on manifolds and metric spaces: what is the probability of a positive definite geodesic exponential kernel? *Proceedings of the 29th Annual Conference on Learning Theory. JMLR: Workshop and Conference Proceedings* **49**, pp. 1647–1650.

Hahn, U. & Jensen, E.B.V. (2016): Hidden second-order stationary spatial point processes. *Scand. J. Stat.* **43**, 455-475.

Hauberg, S., Feragen, A., Enficiaud, R. & Black, M.J. (2016): Scalable robust principal component analysis using Grassmann averages. *IEEE T. Pattern Anal.* **38**, 2298-2311.

Kallenberg, M.G.J., Nielsen, M., Holland, K., Karssemeijer, N., Igel, C. & Lillholm, M. (2016): Learning density independent texture features. *Proceedings of the 13th International Workshop on Breast Imaging. Lecture Notes in Computer Science* **9699**, pp. 299-306. Springer.

Kallenberg, M.G.J., Petersen, P.K., Nielsen, M., Ng, A.Y., Diao, P., Igel, C., Vachon, C.M., Holland, K., Winkel, R.R., Karssemeijer, N. & Lillholm, M. (2016): Unsupervised deep learning applied to breast density segmentation and mammographic risk scoring. *IEEE T. Med. Imaging* **35**, 1322-1331.

Karlsson, P., Haroutounian, S., Polydefkis, M., Nyengaard, J.R. & Jensen, T.S. (2016): Structural and functional characterization of nerve fibres in polyneuropathy and healthy subjects. *Scand. J. Pain* **10**, 28-35.

Kasenburg, N., Darkner, S., Hahn, U., Liptrot, M.G. & Feragen, A. (2016a): Structural parcellation of the Thalamus using shortest-path tractography. *Proceedings of the International Symposium on Biomedical Imaging (ISBI)*, 559-563.

Kasenburg, N., Liptrot, M., Reislev, N.L., Garde, E., Nielsen, M. & Feragen, A. (2016b): Supervised hub-detection for brain connectivity. *Proceedings of SPIE Medical Imaging* **9784**.

Kasenburg, N., Liptrot, M., Reislev, N.L., Ørting, S.N., Nielsen, M., Garde, E. & Feragen, A. (2016c): Training shortest-path tractography: automatic learning of spatial priors. *NeuroImage* **130**, 63-76.

Kasenburg, N., Pedersen, M.V. & Darkner, S. (2016d): Finding significantly connected voxels based on histograms of connection strengths. *Proceedings of SPIE Medical Imaging* **9784**.

Khan, A.R., Chuhutin, A., Wiborg, O., Kroenke, C.D., Nyengaard, J.R., Hansen, B. & Jespersen, S.N. (2016): Biophysical modeling of high field diffusion MRI demonstrates micro-structural aberration in chronic mild stress rat brain. *NeuroImage* **142**, 421-430.

Kjølby, B.F., Khan, A.R., Chuhutin, A., Pedersen, L.H., Jensen, J.B., Jakobsen, S., Zeidler, D., Sangill, R., Nyengaard, J.R., Jespersen, S.N. & Hansen, B. (2016): Fast diffusion kurtosis imaging of fibrotic mouse kidneys. *NMR Biomed.* **29**, 1709-1719.

Kousholt, A. & Kiderlen, M. (2016): Reconstruction of convex bodies from surface tensors. *Adv. Appl. Math.* **76**, 1-33.

Lavancier, F. & Møller, J. (2016): Modelling aggregation on the large scale and regularity on the small scale in spatial point pattern datasets. *Scand. J. Stat.* **43**, 587-609.

Liptrot, M.G. & Lauze, F.B. (2016): Rotationally invariant clustering of diffusion MRI data using spherical harmonics. *Proceedings of SPIE Medical Imaging* **9784**.

Michalkiewicz, M.D., Pai, A., Leung, K.K., Sommer, S., Darkner, S., Sørensen, L., Sporring, J. & Nielsen, M. (2016): Combining the boundary shift integral and tensor-based morphometry for brain atrophy estimation. *Proceedings of SPIE Medical Imaging* **9784**.

Mrkvička, T., Soubeyrand, S., Myllymäki, M., Grabarnik, P. & Hahn, U. (2016): Monte Carlo testing in spatial statistics, with applications to spatial residuals. *Spat. Stat.* **18**, 40-53.

Møller, J., Ghorbani, M. & Rubak, E. (2016): Mechanistic spatio-temporal point process models for marked point processes, with a view to forest stand data. *Biometrics* **72**, 687-696.

Møller, J. & Rubak, E. (2016): Functional summary statistics for point processes on the sphere with an application to determinantal point processes. *Spat. Stat.* **18**, 4-23.

Møller, J, Safavimanesh, F. & Rasmussen, J.G. (2016): The cylindrical *K*-function and Poisson line cluster point processes. *Biometrika* **103**, 937-954.

Pai, A.S.U., Sommer, S.H., Sørensen, L., Darkner, S., Sporring, J. & Nielsen, M. (2016): Kernel bundle diffeomorphic image registration using stationary velocity fields and Wendland basis functions. *IEEE T. Med. Imaging* **35**, 1369-1380.

Pai, A.S.U., Sporring, J., Darkner, S., Dam, E.B., Lillholm, M., Jørgensen, D., Oh, J., Chen, G., Suhy, J., Sørensen, L. & Nielsen, M. (2016): Deformation-based atrophy computation by surface propagation and its application to Alzheimer's disease. *J. Med. Imag.* **3**, 014005.

Rafati, A.H. , Safavimanesh, F. , Dorph-Petersen, K.-A., Rasmussen, J.G., Møller, J. & Nyengaard, J.R. (2016): Detection and spatial characterization of minicolumnarity in the human cerebral cortex. *J. Microsc.* **261**, 115-126.

Rafati, A.H., Ziegel, J.F., Nyengaard, J.R. and Jensen, E.B.V. (2016): Stereological estimation of particle shape and orientation from volume tensors. *J. Microsc.* **261**, 229-237.

Reinert, L.S., Lopušná, K., Winther, H., Sun, C., Thomsen, M.K., Nandakumar, R., Mogensen, T.H., Meyer, M., Vægter, C., Nyengaard, J.R., Fitzgerald, K.A. & Paludan, S. (2016): Sensing of HSV-1 by the cGAS-STING pathway in microglia orchestrates antiviral defence in CNS. *Nature Communications* **7**, 13348.

Rønn-Nielsen, A. & Jensen, E.B.V. (2016): Tail asymptotics for the supremum of an infinitely divisible field with convolution equivalent Lévy measure. *J. Appl. Prob.* **53**, 244-261.

Sander, B. & Golas, M.M. (2016): Molecular electron microscopy: an approach to study macromolecular assemblies. In *Neuromethods* **115**, pp. 205-216. Springer.

Sommer, S. (2016): Anisotropically weighted and nonholonomically constrained evolutions on manifolds. *Entropy* **18**, 425-445.

Sørensen, L., Igel, C., Hansen, N.L., Osler, M., Lauritzen, M., Rostrup, E. & Nielsen, M. (2016): Early detection of Alzheimer's disease using MRI hippocampal texture. *Hum. Brain Mapp.* **37**, 1148-1161.

Waagepetersen, R.P., Guan, Y., Jalilian, A. & Mateu, J. (2016): Analysis of multispecies point patterns using multivariate log-Gaussian Cox processes. *J. Roy. Stat. Soc. C* **65**, 77-96.

CSGB VISITORS - 2016

Gilles Bonnet (Universität Osnabrück, Germany)
31 January – 19 February 2016

Christian Hirsch (Weierstrass Institute, Germany)
8 – 12 February 2016

Fredrik Lindgren (Chalmers University, Sweden)
7 March 2016

Florian Pausinger (TU Munich, Germany)
7 – 11 March 2016

Christoph Thäle (Ruhr Universität Bochum, Germany)
4 – 8 April 2016

Emilio Porcu (University Federico Santa Maria, Chile)
8 – 13 May 2016

Julia Hörrmann (Ruhr Universität Bochum, Germany)
5 – 7 June 2016

Wilfrid Kendall (University of Warwick, United Kingdom) | 5 – 8 June 2016

Viktor Beneš (Charles University, Prague, Czech Republic) | 5 – 10 June 2016

Daryl Daley (University of Melbourne, Australia)
5 – 10 June 2016

Jiří Dvořák (Charles University, Prague, Czech Republic) 5 – 10 June 2016

Richard J. Gardner (Western Washington University, USA) | 5 – 10 June 2016

Daniel Hug (Karlsruhe Institute of Technology, Germany)
5 – 10 June 2016

Günter Last (Karlsruhe Institute of Technology, Germany) | 5 – 10 June 2016

Werner Nagel (Friedrich-Schiller-Universität, Jena, Germany) | 5 – 10 June 2016

Michaela Prokešová (Charles University, Prague, Czech Republic) | 5 – 10 June 2016

Jan Rataj (Charles University, Prague, Czech Republic)
5 – 10 June 2016

Marie-Colette van Lieshout (CWI, Amsterdam, Netherlands) | 5 – 10 June 2016

Wolfgang Weil (Karlsruhe Institute of Technology, Germany) | 5 – 10 June 2016

Adrian Baddeley (Curtin University, Perth, Australia)
7 – 10 June 2016

Michael A. Klatt (Karlsruhe Institute of Technology, Germany) | 10 – 16 June 2016

Alexis Arnaudon (Imperial College, London, United Kingdom) | 18 – 22 June 2016

Tomáš Mrkvička (University of South Bohemia, Czech Republic) | 19 – 26 June 2016

Kassel Liam Hingee (University of Western Australia, Australia) | 20 – 26 June 2016

Yongtao Guan (University of Miami, USA)
1 – 10 July 2016

Joseph Yukich (Lehigh University, USA)
13 – 19 August 2016

Xavier Pennec (INRIA Sophia-Antipolis, France)
13 – 20 August 2016

Anuj Srivastava (Florida State University, USA)
14 – 18 August 2016

Adam Duncan (Florida State University, USA)
14 – 19 August 2016

Benjamin Eltzner (Georg-August-University of Göttingen, Germany) | 14 – 19 August 2016

Tom Fletcher (University of Utah, USA)
14 – 19 August 2016

Julian Grote (Ruhr University Bochum, Germany)
14 – 19 August 2016

Junpyo Hong (University of North Carolina, USA)
14 – 19 August 2016

Bo Markussen (University of Copenhagen, Denmark)
14 – 19 August 2016

Mathew Penrose (University of Bath, United Kingdom)
14 – 19 August 2016

Stephen M. Pizer (University of North Carolina, USA)
14 – 19 August 2016

Anders Rønn-Nielsen (University of Copenhagen, Denmark) | 14 – 19 August 2016

Matthias Schulte (University of Bern, Switzerland) | 14 – 19 August 2016

Anastasios Sidiropoulos (Ohio State University, USA) | 14 – 19 August 2016

Vijay Sridhar (Ohio State University, USA)
14 – 19 August 2016

Jonathan Taylor (Stanford University, USA)
14 – 19 August 2016

Christoph Thäle (Ruhr Universität Bochum, Germany) | 14 – 19 August 2016

Xiaoying Tian (Stanford University, USA)
14 – 19 August 2016

Thomas Hotz (TU Ilmenau, Germany)
14 – 20 August 2016

Hans-Georg Müller (University of California, USA) | 14 – 20 August 2016

Jane-Ling Wang (University of California, USA) | 14 – 20 August 2016

Dhandapani Yogeshwaran (Indian Statistical Institute, India) | 14 – 20 August 2016

Yu Yue (City University of New York, USA)
14 – 20 August 2016

Stephan Huckemann (Georg-August-University of Göttingen, Germany)
15 – 19 August 2016

Steve Marron (University of North California)
15 – 19 August 2016

Marie-Colette van Lieshout (Centre for Mathematics and Computer Science, Amsterdam, Holland) | 5 – 8 September 2016

Aila Särkkä (Chalmers University of Technology, Göteborg, Sweden)
6 – 7 September 2016

Anders Rønn-Nielsen (University of Copenhagen, Denmark) | 26 – 28 October 2016

Ullrich Köthe (University of Heidelberg, Germany) | 4 – 5 November 2016

Sarang Joshi (University of Utah, USA)
12 December 2016

SELECTED CSGB SEMINARS

11 February 2016 | Christian Hirsch (Weierstrass Institute, Berlin): **Stationary Apollonian packings**

18 February 2016 | Gilles Bonnet (Universität Osnabrück): **Cells with many facets in a random hyperplane mosaic**

8 March 2016 | Florian Pausinger (TU Munich): **Functions of bounded variation in numerical integration**

7 April 2016 | Christoph Thäle (Ruhr-Universität Bochum): **Random polytopes and phenomena in high dimensions**

6 September 2016 | Marie-Colette van Lieshout (CWI, Amsterdam): **Likelihood based inference for partially observed renewal processes**

6 September 2016 | Aila Särkkä (Chalmers University of Technology, Göteborg): **Estimating deformation of polar ice**

23 November 2016 | Christophe A.N. Biscio (Department of Mathematical Sciences, Aalborg University): **The accumulated persistence function, a new functional summary statistic for topological data analysis, with a view to brain artery trees and spatial point process applications**

APPENDIX

CSGB SCIENTIFIC STAFF

PROFESSORS

Eva B. Vedel Jensen (EBVJ)
Jesper Møller (JM)
Mads Nielsen (MN)
Jens R. Nyengaard (JRN)
Rasmus P. Waagepetersen (RPW)

ASSOCIATE PROFESSORS

Johnnie B. Andersen (JBA)
Sune Darkner (SD)
Karl-Anton Dorph-Petersen (KADP)
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Monika Golas (MG)
Ute Hahn (UH)
Kristjana Ý. Jónsdóttir (KYJ)
Markus Kiderlen (MK)
François Lauze (FL)
Andrew du Plessis (AP)
Jakob G. Rasmussen (JGR)
Ege Rubak (ER)
Björn Sander (BS)
Jon Sporring (JS)


EBVJ


JM


MN


JRN


RPW


JBA


SD


KADP


AF


MG


UH


KYJ


MK


FL


AP


JGR


ER


BS


JS


CB


ML


SS

ASSISTANT PROFESSORS

Christophe Biscio (CB)

Matthew Liptrot (ML)

Stefan Sommer (SS)


ITA


SH


RJ


AMS

POSTDOCS

Ina Trolle Andersen (ITA)

Stine Hasselholt (SH)

Robert Jacobsen (RJ)

Anne Marie Svane (AMS)


STC


HSC


ADC


JD


RKE


KHJ


AK


LK


AJOM


FACP


AHR


HS

PH.D. STUDENTS

Sabrina Tang Christensen (STC)

Heidi Søgaard Christensen (HSC)

Andreas Dyreborg Christoffersen (ADC)

Jibrin Danladi (JD)

Rikke Krogh Eriksen (RKE)

Katrine Hommelhoff Jensen (KHJ)

Astrid Kousholdt (AK)

Line Kühnel (LK)

Anton Jussi Olavi Mallasto (AJOM)

Francisco Andrés Cuevas Pacheco (FACP)

Ali Hoseinpoor Rafati (AHR)

Helene Svane (HS)


SFB

SOFTWARE DEVELOPER

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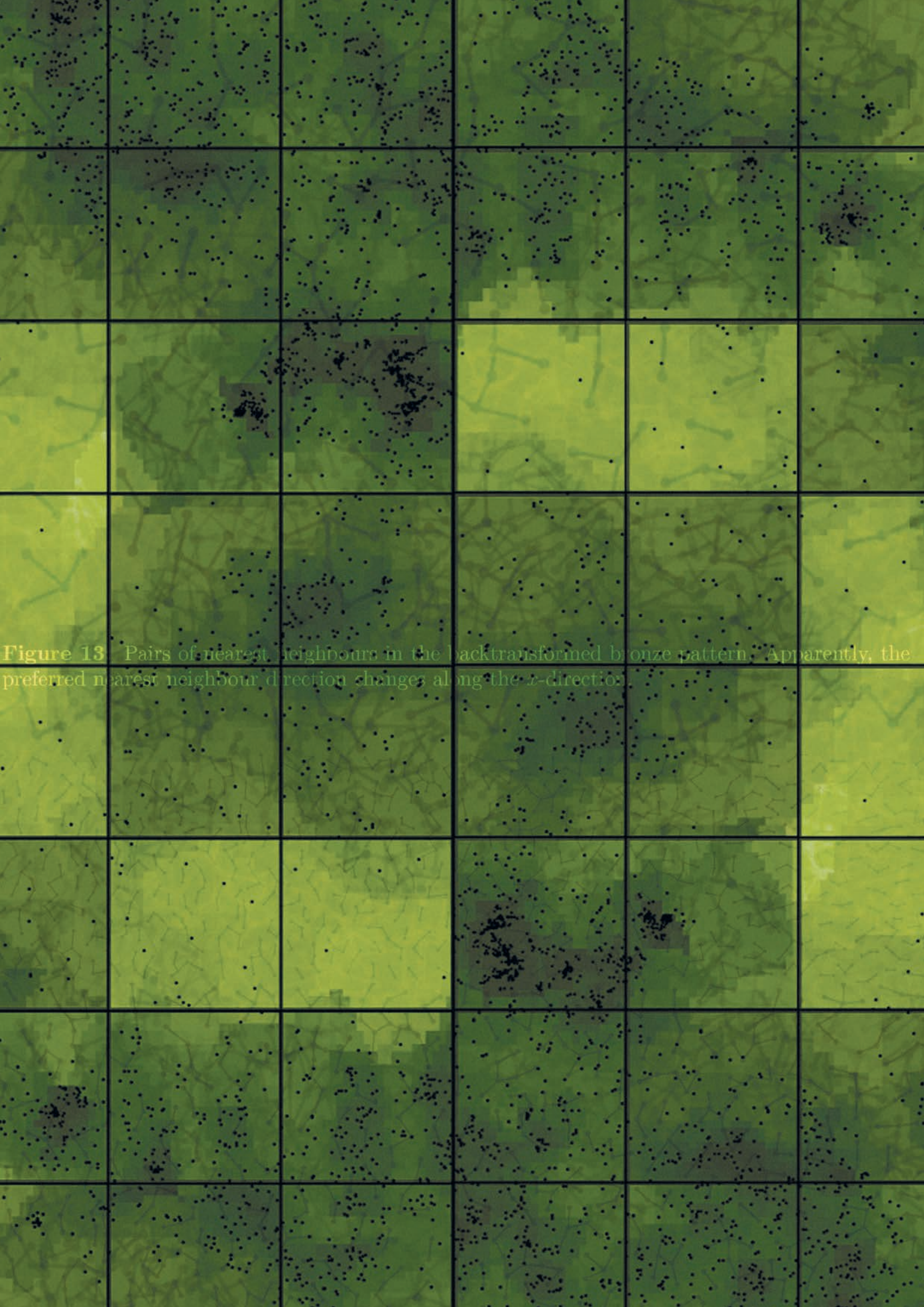

Figure 13. Pairs of nearest neighbours in the backtransformed bronze pattern. Apparently, the preferred nearest neighbour direction changes along the x-direction.

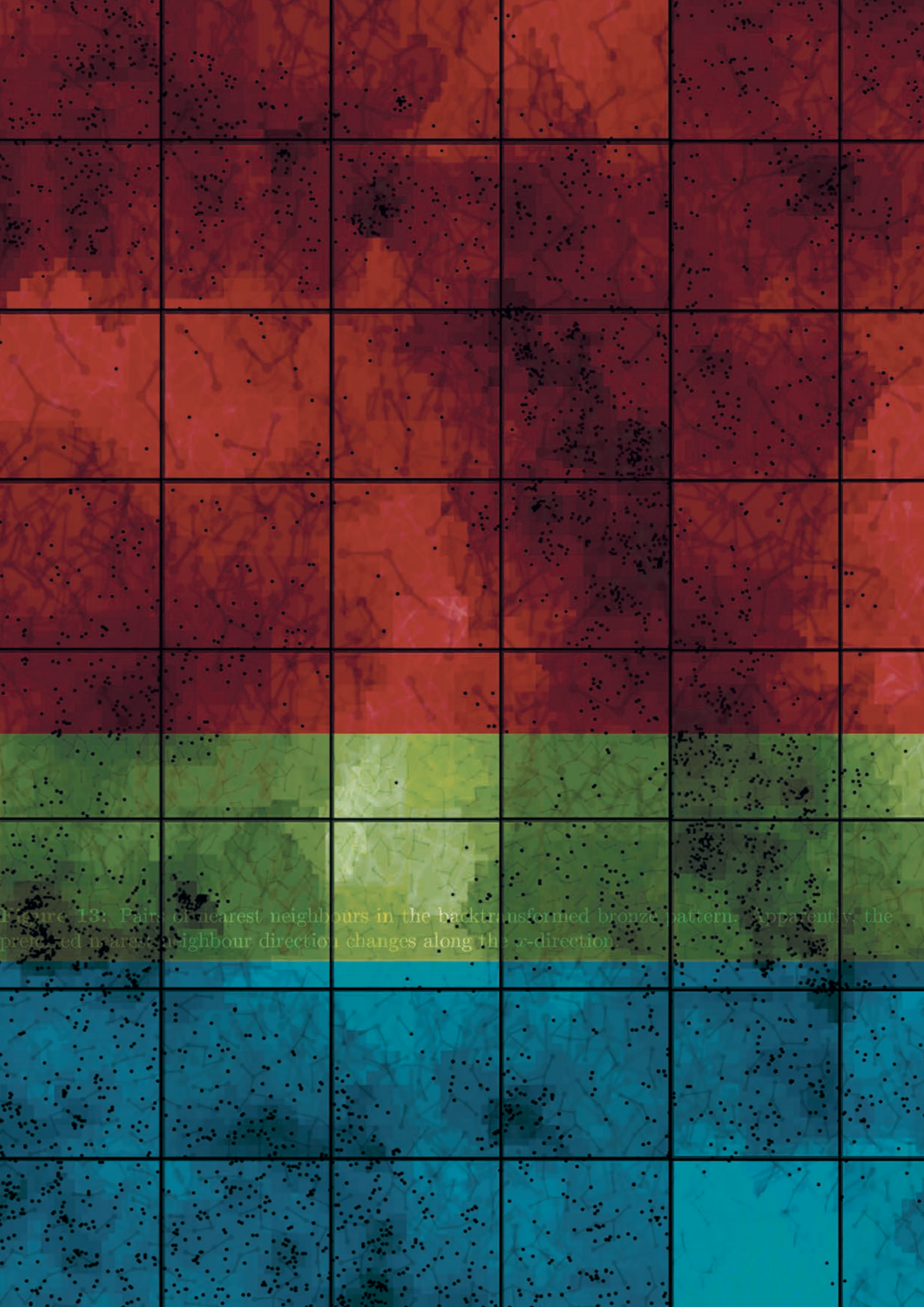

Figure 13: Pairs of nearest neighbours in the backtransformed bronze pattern. Apparently, the preferred nearest neighbour direction changes along the x-direction.